Math 104: Homework 1 (due January 28)

1. Ross exercise 1.3

2. Ross exercise 1.8

3. The Leonardo numbers are defined according to $L_0 = 1$, $L_1 = 1$, and

   $$L_{n+1} = L_n + L_{n-1} + 1$$

   for all $n \in \mathbb{N}$. Consider the function

   $$f(n) = \frac{2}{\sqrt{5}}(\varphi^{n+1} - (1-\varphi)^{n+1}) - 1$$

   where $\varphi = (1+\sqrt{5})/2$ is the Golden Ratio. For $n \in \mathbb{N}$, define P_n to be the proposition that "both $L_n = f(n)$ and $L_{n-1} = f(n-1)$". Apply mathematical induction to prove that P_n is true for all $n \in \mathbb{N}$, and deduce that $L_n = f(n)$ for all $n \in \mathbb{N} \cup \{0\}$.

4. Ross exercise 2.4

5. Show that $\sqrt{2} + \sqrt{3}$ is irrational.

6. Ross exercise 3.1

7. Ross exercise 3.4

8. Show that $||a| - |b|| \leq |a - b|$ for all $a, b \in \mathbb{R}$.

9. Ross exercise 3.6