CALCULUS AND ANALYTIC GEOMETRY

Instructor's Guide

October 25, 2002

Note that department policy requires advance approval for any section deviating significantly from this official syllabus!

Text: Varberg, Purcell, Rigdon: **CALCULUS** (8th Edition) Prentice Hall.

PREREQUISITES:
For 221: Math 112 and 113 or Math 114 or satisfactory placement scores.
For 222: Math 221 or Math 171 and 217.
For 234: Math 222.

General Remarks and Suggestions:

The purpose of this sequence of courses is to train future users of mathematics rather than future mathematicians. The primary emphasis should be on problem solving techniques, with an intuitive understanding of why they work. There is a separate calculus honors sequence, Math 275-276-277. Except in those courses, informal explanations are likely to be more valuable than rigorous proofs. Almost none of the students in the non-honors courses 221-222-234 will identify themselves as math majors, although there will be a number of gifted and highly competent individuals in these courses. It might be appropriate to identify some of these and to encourage them to enroll in the honors sections of subsequent courses.

Students seem to consider the idea that calculus could be interesting and intellectually stimulating to be ridiculous. That belief is unfortunate but is not sufficient reason to dismiss the students as dumb or impossible to teach. We obviously know that mathematics **is** exciting, and should view the student attitude as a challenge to be overcome. "Converting" a student is a real accomplishment. Many students approach these courses by seeking to develop a library of methods for solving standard problems in lieu of thinking. While it may be impossible to eliminate this approach, we urge attempts to reduce it. The use of study guides with many problem solutions should be discouraged.

Course Organization: 221 and 222

These courses are generally given in the lecture-discussion format, while some sections use a "satellite" format. In the lecture-discussion format, the lecturer meets the class three times per week (some sections may have two 75-minute lectures Tuesday and Thursday instead of three 50-minute lectures Monday-Wednesday-Friday), covers the new material, prepares and works illustrative examples and assigns homework problems. The students also meet twice a week in a discussion section taught by a Teaching Assistant. It is in these TA sections that assigned problems are discussed and most questions are answered. (It is, of course, vital that each TA be prepared to work all of the problems.) The TAs are required to attend the lectures and to hold office hours for their students. (Part of the time for which TAs are being paid is the lecture time.) It is essential that lecturers meet periodically with their TAs in order to discuss the progress of the course and to get feedback from them. Generally, the TAs will collect and grade at least some of the students' homework and it is they who, in consultation with the lecturer, assign final course grades to each student. Although the precise responsibilities delegated to the TAs may vary from lecturer to lecturer, it is customary to allow the TA's evaluation of each student, based on homework, quizzes and class participation, to be a component of the final grade.

In the satellite format, a faculty member teaches a section meeting five days per week and supervises TAs who teach their own sections five days per week. The faculty member and the TAs stay on the same schedule and give common exams. There **must** be close coordination of these sections: By agreeing to teach a satellite section, a faculty member is implicitly agreeing to carry out the supervision and coordination necessary to make this format work. Typically this will require weekly meetings involving the faculty member and all of the TAs working with him or her. The schedule data below do not apply exactly to the satellite format since the lecture/discussion distinction is gone, but the material covered must still be the same.

Course Organization: 234

234 is generally taught in a lecture-discussion format where each student is registered for three lectures and one discussion section each week. It was originally intended that the lecture have 80 students and each discussion section have 20, but in fact a lecture more typically has between 160 and 220 students. Compared with 221 and 222, you will probably find that you have more than enough lecture time to cover the minimum material, but that you must expect less of the teaching assistants.

The course should begin with a quick review of vectors even though this is part of the syllabus of 222. Many transfer students (even from other schools in the UW system) will be placed in 234 after having had a calculus course not including vectors but otherwise comparable with 222.

The role of the TA in 234 is clearly different from that in 221 and 222, and how the lecturer and TA can be most effective has required some thought and experiment. Here are some ideas which may help. The TA is typically being employed to teach 4 sections, each of about 20 students and meeting one 50-minute hour per week. Two of his/her sections will meet after one of your lectures and the other two after a different lecture. For that reason, the TA will not be able to play much of a role in filling in material which may have been omitted in lecture. (That should not be a problem, since 234 covers material at a more leisurely pace than 221 or 222.) In the department's work agreement with the TAs, the TA is being paid for 90 hours of grading during the semester. This is 5-6 hours per week, so it is quite reasonable to expect the TAs to grade some homework or quizzes. (This time would cover about half the grading they'd be expected to perform in 221 or 222, and includes grading of other items such as exams.)

With four discussion sections (up to 100 students) to meet, and fewer quizzes or homework assignments to base classwork grades on, it is easy for the TA to lose involvement with his/her students. Here are some suggestions to get students working at the beginning of the semester, and to keep them showing up for discussion sections:

1. Give an early first exam, which covers material on vectors which may not have been covered in 222.

2. Make one or more individual projects part of the course. This might involve writing up careful proofs of the first or second Kepler laws. Other ideas are given in various books on student projects in calculus.

3. Ask the TAs to take attendance fairly regularly, and have them let their students know that this record of attendance could make a difference in final grades for borderline cases.

Grading and Other Responsibilities

The lecturer or satellite leader is responsible for supervising the TAs and evaluating their performance. In addition to a two hour final exam written by the lecturer or satellite supervisor, there should be at least two midterm exams.

The exams should consist largely of problems comparable in difficulty to the assigned homework. Include one or two more difficult problems which require a greater depth of understanding and some original thought, but resist the temptation to give a problem so interesting that no student can do it. Many instructors feel that students should be able to give coherent definitions and even write easy proofs (e.g. $(uv)' = u'v + uv'$); the latter is feasible if you ask a proof from a list of proofs that the students have been told to prepare.

Because there are barely enough lectures to complete the following syllabi, these exams may be given in the evening. This has the advantage that longer time slots can then be used. (We recommend that students be given at least 90 minutes to show what they can do.) In order to maximize uniformity of grading of the exams, the usual procedure is for the lecturer and all of the TAs to get together for a joint grading session. Each problem is then graded either by a single individual or by several people who have agreed on a common scheme for assigning partial credit. It is a good idea for the lecturer to establish these partial credit assignment schemes. TAs should be instructed to write helpful comments on tests when they deduct points and to deduct points for bad or unclear exposition and any false statements, *even if the final answer is correct.*

Much discussion time can be saved if exam solutions are distributed at the end of the test. The preparation of the solution sheet also gives a good idea about the length and difficulty of the exam. The last step in the grading process, usually done by the lecturer and TAs together, is the establishment of an appropriate "curve". To insure some uniformity of grading from lecture to lecture and to prevent "grade inflation", we propose that about 15% of the students get As and ABs and that the median grade be a BC. (While the quality of students may vary significantly from section to section, it seems fairly safe to assume that a large lecture or the union of the satellite sections will be approximately "average" within some population. The populations differ with time, however: Generally in the spring semester 221 students are apt to be weaker than in the fall; fall sections of 222 include a mixture of advanced placement freshmen, apt to be very good, and students who either failed 221 before or had to take precalculus courses. In recent years the advanced placement group has frequently been a majority.)

Students often want to know exactly what they have to write on an exam to prove that they understand. This is a difficult political issue, and you should think carefully about how you respond to this issue and how you word an exam question to be consistent with your response.

Timing and Content

It is essential to follow these syllabi closely. Students normally take these courses to meet requirements which assume certain material is learned. Students also switch from one instructor to another between 221 and 222, or between 222 and 234. There is little extra time and every effort should be made not to fall behind schedule. The material at the ends of these courses should not be sacrificed for the sake of a more extended treatment of earlier topics. Lecturers should be careful in monitoring progress and the number of lectures remaining to avoid running out of time, and be ruthless in resisting the temptation to exceed the budgeted time. Some topics could be assigned as outside reading. Learning on their own is an important skill for students to develop and it is not a bad idea for them to be held responsible for material not covered directly in class, but they have probably never had that responsibility so they deserve warning.

A typical semester has 15 weeks but usually the first Monday is a holiday (Labor Day in the fall and Martin Luther King Day in the spring) and in the fall the Friday following Thanksgiving is a holiday. Thus there will be 43 MWF lectures in a typical semester. A lecturer should always allow at least one discussion section between any lecture introducing new material and any exam; in particular, new material should not be covered in the last lecture. The Wednesday before Thanksgiving and the Friday before spring break may not be suitable for covering important new material. Effectively, this mean that *the lecturer should say everything s/he has to say in 40 lectures.* The approximate schedules below must be adjusted proportionally for a TR schedule, which will include approximately the same lecture time.

The suggested schedules below go through the text *almost* in order. Students frequently find it confusing to jump around in the text in another order, and doing so requires extreme care in selecting problems to assign so as to avoid using material not yet covered. The one major deviation from covering the chapters in order is the insertion of chapter 18 on Differential Equations in 222.

Math 221 (Starting Fall 2002)

You should cover the first 7 chapters of the official text, Varberg, Purcell, Rigdon: *Calculus*, Eighth Edition. The schedule below shows a rough schedule for completing the course in the allotted time. Remember that teachers of subsequent courses will assume that students have seen this material. As long as you adhere to this constraint, there is no need to follow the schedule given below religiously. Some of the topics listed as optional (such as precise treatment of limits, related rates, work, and hyperbolic functions) may be considered absolutely essential by some instructors. It is certainly possible to cover some of them, but **NOT** at the expense of topics identified as central.

Week 1

1.1-1.8, 2.1-3 *Review of Precalculus.* Many students will need this review, but if you spend to much time on it, you will run out of time at the end of the semester. Remember that your students have shown, on the placement test, that they can do most of this material under some circumstances. Of course they will deny it if asked; to admit it would be to invite being held responsible for it. So go ahead and hold them responsible for it and you may be surprised how many will rise to your desires rather than fall to your expectations. Review Cartesian coordinates, the distance formula (Pythagorean Theorem), and equations for lines and circles. Advertise the three review sessions run by the tutorial program early in the semester. You might have the TA's give an algebra skills quiz in the discussion section before the first lecture. Parametric equations are not used in the text until section 6.4 where they are introduced in the section on arclength. You might want to use them more systematically; for example, the parametric equations

$$x = \cos\theta, \qquad y = \sin\theta$$

help students decide the sign of the various trig functions in each quadrant.

2.4-2.6 *Limits.* Most students will not grasp $\varepsilon - \delta$; starting the course with it will create much anxiety. If you want to teach $\varepsilon - \delta$, it might be better to wait until the students have successfully learned something else. Explain that the variable x in $\lim_{x\to a}$ is a dummy variable. Cover the concept of "dummy variables" again when doing sums and definite integrals in chapter 5.

Week 2

2.6-2.10 *Limits and continuity.* Emphasize the definition of continuity and the limit laws and the correct use of notation. Note the theorem

$$\lim_{t\to 0}\frac{\sin t}{t} = 1, \qquad \lim_{t\to 0}\frac{1-\cos t}{t} = 0,$$

appears here as Theorem B on page 78, earlier in Varberg than in other texts like Thomas or Stewart. This is the hard part of the proof that the derivative of the sine is the cosine which is proved in section 3.4.

Students often use the word “exists” incorrectly. On page 85 (problems 48 and 49 for example) the text distinguishes three possible answers to the question “find the limit”, namely (1) there may be a (finite) number L with $\lim_{x\to a} F(x) = L$, (2) either $\lim_{x\to a} F(x) = \infty$ or $\lim_{x\to a} F(x) = -\infty$, and (3) the limit does not exist even in the infinite sense. Students who have had calculus may think that the answer "the limit does not exist" is appropriate in both cases (2) and (3) and you should force them to distinguish these two cases. WARNING: Many students have had calculus in high school and have already learned l’Hôpital’s rule. When you test this material, you may want to word your exam questions to explicitly forbid their using l’Hôpital’s rule.

3.1 *Tangent and Velocity.* This is what calculus is about. Over a small time interval the average velocity is roughly the same as the instantaneous velocity (the derivative). The Varberg text will define the average velocity for a function in section 5.7 (page 255); unfortunately, it does not point out there that the Fundamental Theorem tells us that the average velocity in the sense of 3.1 is the same as the average velocity in the sense of 5.7.

Week 3

3.2 *The Derivative.* Students confuse derivatives and limits. Emphasize that a derivative is a special case of a limit.

3.3 *Differentiation formulas.*

3.4 *Derivatives of trig functions.* The Varberg text has already done the hard part in 2.7.

Week 4

3.5 *Composite functions.* It may be helpful to motivate the chain rule with velocity in miles/hour as compared to miles/minute, and more generally $y = f(x)$, $x = g(t)$ as a change of scale from x-units to ‘t-units’. Do some examples which cannot be done without the chain rule.

3.6-3.7 *Leibniz notation and higher derivatives.* Emphasize correct use of notation. Note the table on page 133 which contains all the common notations for derivatives.

3.8 *Implicit differentiation.* The text does the power law for rational powers here.

Week 5

3.9 *Related Rates.* Related rates is listed as optional, but a small amount of time on this can provide good examples of “story” problems as well as situations where the chain rule is obviously essential.

3.10 *Linear Approximation.* Like most texts the treatment of differentials is confusing. ($\Delta x = dx$!?). It is best just to say that the equation $dy = g(x)\,dx$

is an abbreviation for the equation $\frac{dy}{dx} = g(x)$. Emphasize that the students should not write an infinitesimal on one side of an equation but not on the other. Linearization should be more than a method for computing approximations. The idea that "differentiable functions are almost straight if you just look closely enough" can be assisted by calculator or computer exercises or demonstrations. You can mention linear approximation, quadratic approximation, and hint that this process can be continued; this will plant the seeds that will facilitate (Taylor) series in 222.

Week 6

4.1 *Max Min.*

4.2 *Monotonicity and Concavity.* Note that the proof of the Monotonicity Theorem is postponed till 4.7. As is customary in most modern calculus texts Varberg uses the terms *concave up* and *concave down* instead of *convex* and *concave.*

4.3 *Local Max Min.*

Week 7

4.4 *Max Min word problems.* Students have difficulty setting up the problem, i.e. translating words into formulas. Skip 4.5 (Economic Applications) and the optional material on least squares in 4.4.

4.6 *Curve sketching.* Curve sketching needs motivation in an era of graphing calculators. Graphing calculators can be allowed on exams if graphing questions ask for understanding and not just a picture. For example, you can give a list of properties of a function and ask for a sketch, without specifying a function, or you can give the graph to the students and ask them to extract properties. It is crucial that students know the shape of basic functions: x^n, $x^(-1/n)$, trig, exp, ln,... They also must learn to recognize key features of a function (e.g. singularities).

Week 8

4.7 *Mean Value Theorem.* Here is the proof of the Monotonicity Theorem and the theorem that two functions with the same derivative differ by a constant. If time permits, mention (or even prove) that similar arguments prove "concave up implies tangent line below and secant line above".

Week 9

5.1-5.2 *Antiderivatives and Differential equations.* Emphasize that the formulas $F' = f$ and $\int f(x)\,dx = F(x) + C$ are synonymous and explain that the latter is motivated by the Fundamental Theorem which will be proved later. Do the equation $y'' = -g$ where g is constant and solve some easy differential equations using separation of variables. Emphasize that solving an initial value problem involves evaluating the constant of integration.

5.3 *Sigma Notation.* Make sure the students can use sigma notation correctly and use it (as does the text) to explain Riemann sums. Cover the concept of "dummy variables" while doing sums and definite integrals.

4.4-5.5 *The definite integral (area under a curve).* Emphasize that a Riemann sum approximates the integral. Consider assigning homework questions like "Find a Riemann sum bigger than the integral ..." where the integrand is complicated or where only partial information is given about the integrand.

> *The last day when students may drop a course typically falls in the 9th or 10th week. It is a good idea that the weak students know who they are before this.*

Week 10 (Mar 31- Apr 5)

4.4-5.5 *The definite integral (area under a curve).*

5.6-5.7 *The Fundamental Theorem.* Point out that the Fundamental Theorem tells us that the average velocity in the sense of 3.1 is the same as the average velocity in the sense of 5.7. Skip the Mean Value Theorem for Integrals.

Week 11

5.8 *Evaluating Integrals.* Note that Example 7 on page 253 in section 5.7 uses substitution although this is not justified till section 5.8. Be sure to cover substitutions with definite integrals (Theorem B on page 260), and get students to use this method rather than do all substitutions with indefinite integrals.

6.1-6.? *Applications of the integral.* Skip work or center of mass rather than run out of time to treat chapter 7 well. (Both these applications are treated again in 234.) Arclength (section 6.4) is important because it is the first point in the text where parametric equations are used.

Week 12

7.1 *The natural log.*

7.2 *Inverse Functions.* The view of the text is that the graphs $y = f(x)$ (suitably restricted) and $x = f^{-1}(y)$ are exactly the same. This is better than what most texts do; namely, to tell the students to graph $y = f^{-1}(x)$ by interchanging x and y in $y = f(x)$ and then solving for x in terms of y. (In general, label the axes when you draw a graph and don't confine yourself to the case where the horizontal axis is the x-axis and the vertical axis is the y-axis.

7.3 *The exponential function.*

7.4 *Logs and exponentials base a.*

> *Thanksgiving recess typically falls in the 12th or 13th week in the fall.*

Week 13

7.5 *Exponential growth.* The equation $y' = ky$.

7.6 *First order linear ODE.* The equation $y' + P(x)y = Q(x)$. In 7.5 and 7.6 you can use differential notation (like is done in science courses) to derive differential equations governing exponential growth and mixtures.

Week 14

7.7 *Inverse trig functions.*

7.8 *Hyperbolic functions.* This is optional. You could introduce $e^{i\theta} = \cos\theta + i\sin\theta$ as motivation, but the text never uses this notation. In any case, point out that the solutions of $y'' + y = 0$ are linear combinations of sine and cosine and the solutions of $y'' - y = 0$ are linear combinations of sinh and cosh.

Week 15

9.1-9.2 *Indeterminate forms (l'Hôpital's Rule).* You needn't prove l'Hôpital's Rule as in the book (i.e. using the Cauchy Mean Value Theorem) but might just do the easy case where the denominator has a non vanishing derivative. You might want to do these topics earlier in the semester, but beware of the fact that some problems use logs and exponentials and should not be assigned before Chapter 7 is studied.

Math 222 (Starting Spring 2003)

Math 222 is divided into four parts: formal integration, infinite series, differential equations, and analytic geometry. Like 221 it is overcrowded; unlike 221 there is no unifying theme. Here are some general comments on each of the four parts.

1. Formal Integration. Most of the integrals done here can be done by symbolic computation programs like `Maple` or `Mathematica`. The primary purpose should be to improve algebra skills. Reinforce understanding of the Fundamental Theorem and changing the limits of integration (Theorem B on page 260) by giving some definite integrals as problems.

2. Infinite Series (including improper integrals). Students find this the most difficult topic in the entire course, and neither they nor the departments who require them to take the course generally see any need for very formal treatment of series. Power series and approximation of functions are the only topics in this chapter which most other departments find useful, and the only topics we ourselves ask the students to use in any detail in courses following soon after calculus. Students who need more detailed knowledge of series, including fluency with convergence tests, will be taking higher level courses where they can learn the real mathematics involved.

3. Differential Equations. We used to cover this topic in Math 223 where we allotted five weeks to it. After extensive negotiations with the College

of Engineering we added some differential equations to Math 222 and replaced Math 223 (a five credit course) by Math 234 (a three credit course). There is little time to do more than second order linear equations with constant coefficients. First order linear equations should be treated in Math 221, but a quick review in Math 222 is appropriate.

4. Analytic Geometry (including vectors). The choice of material taught should be governed by the objective of ending the course with planetary motion, i.e. the derivation of Kepler's laws from Newton's Third Law and Newton's Inverse Square Law (or the reverse). To our knowledge, the only other place in the curriculum where this is taught is Physics 311 a course which attracts only seventy students per year.

The following schedule shows how you can cover all this in a semester.

Week 1

8.1 *Integration is Antidifferentiation.*

8.2 *Integration of Trig Functions.*

8.3 *Integration by Trig Substitution.*

Week 2

8.4 *Integration by Parts.*

8.5 *Integration of Rational Functions.*

Week 3

8.6 *Miscellaneous Problems.*

9.3-4 *Improper Integrals.* Emphasize integrals over unbounded intervals more than integrals of unbounded functions. Point out that the Fundamental Theorem will not work if there is an interior discontinuity. This material is probably worth two lectures: A good understanding of improper integrals can pave the way for series. Convergence and divergence testing for improper integrals can build some understanding of rate of growth of a function. You should do this just before starting series, and emphasize the similarities between convergence of an integral on the positive x-axis and convergence of a series. You can combine the formula

$$\int_a^\infty f(x)\,dx = \int_a^b f(x)\,dx + \int_b^\infty f(x)\,dx$$

with the comparison test to motivate the idea that when we know why an improper integral converges, we can also estimate the error between the definite integral with $b \approx \infty$ and the improper integral.

Week 4

10.3-5 *Convergence Tests.* There is not enough time to cover all convergence tests, nor is it necessary. Comparison, geometric series, and the ratio test are the most important. The Limit Comparison and Integral tests are worthwhile but are candidates to drop if you don't have enough time. If you can fit it into the available time, the integral test (a) builds a connection to improper integrals and (b) can be applied right away to show when p-series converge. If you do not do the integral test you will probably want to include the facts on p-series anyway, perhaps just as "we don't have the tools to show this but here is what happens", so that they can be used for comparisons.

You can combine the formula

$$\sum_{k=0}^{\infty} a_k = \sum_{k=0}^{N} a_k + \sum_{k=N+1}^{\infty} a_k$$

with the convergence tests to motivate the idea that when we know why an infinite series converges, we can also estimate the error between the finite sum with $N \approx \infty$ and the infinite sum. Emphasize that absolute convergence implies convergence. If you do alternating series, be sure to explain that Theorem A on pages 453-4 shows that the next term gives an error estimate.

Week 5

10.6-7 *Power Series and Taylor Series.* Stress that a function is not always equal to its Taylor series, but that where it *is* analytic (this term is *not* used in the text) one can operate on the function by operating on the series term by term. Point out how this can simplify computation of a Taylor series, e.g. by comparing direct calculation of the series for $\sin(x)/x$ to reducing all of the exponents in the series for $\sin(x)$ by 1.

Week 6

10.8,11.1 *Taylor's Formula - Estimating the Error.* Note that Lagrange's form for the remainder is proved in 10.8 and used in 11.1. Theorem C in 10.8 states that a function is analytic iff $\lim_{n\to\infty} R_n(x) = 0$. On page 434 in 11.1 there is some discussion of computational error.

Week 7

7.6,18.1 *Differential Equations.* Quickly review section 7.6 in case students have never had (or have forgotten) it.

Week 8

18.1 *Linear Homogeneous.* Emphasize second order with constant coefficients. You can do higher order with constant coefficients as preparation for the method of undetermined coefficients.

18.2 *Linear Inhomogeneous.* The text does both the method of undetermined coefficients (which works when the inhomogeneous term is the root of a homogeneous linear differential equation with constant coefficients) and the method of variation of parameters (which works when you can do the integral that results).

Week 9

18.3 *Applications.* If you have time do exercise 14 on page 785 illustrating resonance (and bridge collapse?).

The last day when students may drop a course typically falls in the 9th or 10th week. It is a good idea that the weak students know who they are before this.

Week 10

12.1-3 *Conic Sections.* The text defines a conic section as a section of a cone and uses the focus-directrix definition after asserting the equivalence. If you want to prove the equivalence of these definitions be warned that the picture is very hard to draw on the blackboard. The text then shows that the focus-focus definition of the ellipse is the same and describes the optical properties. (Very nice.) Some of these pictures can be nicely done using a computer. Bob Wilson has a Maple routine that lets you tip the slicing plane back and forth and see what the intersection looks like.

12.4-5 *Changing Coordinates.* This is done completely (unlike some texts) and is a central topic.

Week 11

12.6-7 *Polar Coordinates.* You will need the polar equation for the ellipse to do planetary motion.

12.8 *Calculus in Polar Coordinates.* You will need the formula $dA = \frac{1}{2}r^2\,dr\,d\theta$ to do planetary motion.

Week 12 The material in Chapters 13 and 14 can be done simultaneously: vectors in the plane are a special case of vectors in space.

13.1 *Parametric Equations.*

13.1-3,14.1-2 *Vectors.* The text (like most texts) uses $\overrightarrow{AB}$ and $\mathbf{v}$ to denote vectors: The arrow notation is used only for the representation of a vector by a directed line segment. In lecturing it is customary to write $\overrightarrow{V}$ rather than $\mathbf{v}$. Point out the need for a handwritten symbol replacing bold-faced type. Distinguish between points P and vectors $\mathbf{v}$ and indicate that the choice of an origin O makes points and vectors correspond via the radius vector $\mathbf{r} = \overrightarrow{OP}$. Note that the text uses lower case boldface for vectors and writes $P = (x, y, z)$ for points rather than $P(x, y, z)$ as in some texts. The text introduces the notation $\langle a, b\rangle$ as synonymous with $a\mathbf{i} + v\mathbf{j}$ on page 571. The notation $\langle a, b\rangle$ is not heavily used (to my knowledge) in client departments. Emphasize that these are two different notations for the same thing. On page 571 the book points out that (a, b) is a point and $\langle a, b\rangle$ is a vector.

Week 13

13.3,14.2-3 *Dot Product and Cross Product.*

Thanksgiving recess typically falls in the 12th or 13th week in the fall.

Week 14

14.4 *Lines and Planes.*

13.4-5,14.5 *Velocity, Acceleration, and Curvature.* Include the material from these sections which is necessary for your treatment of the Kepler problem.

Leave sections 14.6 (surfaces in three space) and 14.7 (polar and spherical coordinates) for Math 234.

Week 15

Notes *The Kepler Problem.* The present text does not cover this material, but many other calculus texts (e.g. Thomas Finney Fifth) do. Several department members have produced their own notes on the topic. See for example

`http://www.math.wisc.edu/~passman/planet.pdf` or
`http://www.math.wisc.edu/~robbin/234dir/kepler.pdf`

(Tell the Calculus Committee if you have notes you are willing to contribute.)

Math 234 (Starting Fall 2003)

234 is a *three* credit course, unlike 221 and 222. As such it will meet fewer times in the semester, but on the other hand the students who have finished 222 and continue to 234 are generally more mathematically able as well as more advanced in study skills. Thus they can be expected to take a larger role in working through new material. In addition the smaller(?) classes allow for more effective use of class time. The schedule below assumes two hours per week are devoted to new material and one to discussion, testing, etc.

Week 1

14.1-5 *Review of vectors.* Many instructors do not like to start the semester with a review. However, it is probably appropriate here since transfer students (even those from other campuses in the UW system) are often placed in Math 234 after having had a calculus course not covering vectors but otherwise comparable to our Math 222.

13.5,14.5 *Velocity, acceleration, and curvature.* Do these since curvature was optional in Math 222. The binormal **B** is defined on page 616. You might have time to do the Frenet equations

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N}, \qquad \frac{d\mathbf{N}}{ds} = -\kappa\mathbf{N} - \tau\mathbf{B}, \qquad \frac{d\mathbf{B}}{ds} = \tau\mathbf{N},$$

but these are not covered in the text.

Week 2

14.6 *Surfaces in three-space.* You could discuss parametric surfaces

$$x = x(s,t), \qquad y = y(s,t), \qquad z = z(s,t),$$

although the text only treats them as part of a "technology project" at the end of Chapter 17.

14.7 *Cylindrical and Spherical coordinates.* The sphere of radius a has the parametric equations

$$x = a\cos\theta\sin\phi, \qquad y = a\sin\theta\sin\phi, \qquad z = a\cos\phi.$$

Note that the text specifies spherical coordinates in the order (ρ, θ, ϕ) while some others use (ρ, ϕ, θ) and some books use entirely different names for the variables.

Week 3

15.1 *Multivariate functions.*

15.2 *Partial derivatives.* Students often have difficulty using differentiation rules for one variable in calculating partial derivatives.

Week 4

15.3 *Continuity.* Do not go into all the nuances of limits of functions of several variables. Do point out that continuity with respect to each variable separately does not suffice for continuity of a multi-variable function.

15.4 *Differentiability.* Here the key formula is

$$f(P + \mathbf{h}) = f(P) + \nabla f(P) \cdot \mathbf{h} + \varepsilon(\mathbf{h}) \cdot \mathbf{h}$$

on page 652. There is a typo on page 652: $\langle f_x(P_0), f_y(P_0)\rangle \cdot \mathbf{h}$ should appear where $(f_x(P_0), f_y(P_0)) \cdot \mathbf{h}$ appears. (See page 571.) It is probably better to write $a\mathbf{i} + b\mathbf{j}$ rather than $\langle a, b\rangle$ to better emphasize the distinction between points and vectors.

Week 5

15.5 *Directional derivatives and gradients.*

15.6 *Chain rule.*

Week 6

15.7 *Tangent planes and linear approximation.* Emphasize local linearization, the idea being that the surface described by a differentiable function is "almost a plane (the tangent plane) if you just look closely enough." This idea motivates the chain rules for several variables and directional derivatives. The text uses the notation $\langle x - x_0, y - y_0, z - z_0\rangle$ rather than $(x - x_0)\mathbf{i} + (y - y_0)\mathbf{j} + (z - z_0)\mathbf{k}$ on page 667. The notation $\langle x - x_0, y - y_0, z - z_0\rangle$ was introduced on page 571 (in Math 222) but is not heavily used (to my knowledge) in client departments.

15.8 *Max and min.* You can relate this to rotation of the axes by studying functions of form

$$f(x, y) = A(x - x_0)^2 + B(x - x_0)(y - y_0) + C(y - y_0)^2$$

and describing the level curves.

Week 7

15.9 *Lagrange multipliers.* Some departments would like us to give a lot of attention to constrained max/min and Lagrange multipliers. You may not have time to do that. If you do cover constrained max/min, however, emphasize problems for regions with boundaries, using Lagrange multipliers for the boundary analysis. Confine your attention to problems with one constraint.

Week 8

16.1-2 *Double integrals and iterated integrals.*

Week 9

16.3 *Integrals over nonrectangular regions.*

16.4 *Polar coordinates.*

> *The last day when students may drop a course typically falls in the 9th or 10th week. It is a good idea that the weak students know who they are before this.*

Week 10

6.6,16.5 *Center of mass.* The centroid (by definition) is the center of mass when the mass distribution is uniform. Figure 10 on page 308 says that "the center of mass is the center of mass of the centers of mass." This can now be explained as special case of the formula for computing a two or three dimensional centroid.

16.6 *Surface area.* The right way to do surface area is to parameterize the surface via

$$\mathbf{R} = x(s,t)\mathbf{i} + y(s,t)\mathbf{j} + z(s,t)\mathbf{k}$$

and then do

$$A = \int dA, \qquad dA = \left| \frac{\partial \mathbf{R}}{\partial s} \times \frac{\partial \mathbf{R}}{\partial t} \right| ds\, dt$$

with appropriate limits of integration. The formula in the book is the special case

$$\mathbf{R} = x\mathbf{i} + y\mathbf{j} + f(x,y)\mathbf{k}.$$

The formula $dA = r\,dr\,d\theta$ is the special case

$$\mathbf{R} = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j}.$$

More generally, the change of variables formula in a double integral is also a special case.

Week 11

16.7, 16.8 *Triple integrals.* While multiple integrals may be motivated by volumes, be sure to stress that they need not represent volumes.

Week 12

17.1 *Vector fields.*

17.2 *Line integrals.*

> *Thanksgiving recess typically falls in the 12th or 13th week in the fall.*

Week 13

17.3 *Independence of the path.* Emphasize that the integral

$$\int_C M\,dx + N\,dy + P\,dz$$

is independent of the parameterization of C but that the integral

$$\int_C \frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy + \frac{\partial f}{\partial z}\,dz$$

only depends on the endpoints (i.e. is independent of the path).

17.5 *Surface Integrals.*

Week 14

17.4 *Green's Theorem.*

17.6 *The Divergence Theorem.*

Week 15

17.7 *Stokes Theorem.*

Technology

Graphing calculators are now readily available, and many students will have them, and may find them helpful in learning the subject. Some instructors believe that these calculators should be allowed on exams (and that questions of the sort: "plot the graph of $y = x/(x^2+1)$" are thus not appropriate). These calculators can be programmed to store notes and formulas, and so, in fairness, if they are allowed for exams, every student should be allowed to bring a page of notes. Note that there is little time in the semester to discuss how it can happen that a calculator can give misleading results.

One approach which can help get get students more involved is to assign topics for them to study and write up, possibly in groups. This will work best in Math 234 where there is more time. The MAA book **Student Research Projects in Calculus** by Cohen et al gives some advice on running such projects, as well as some possible topics. The use of projects has been successful when tried; students like them and put some real effort into them. There are many suitable projects suggested in the five-volume collection **Resources for Calculus** published by the MAA; copies may be borrowed in 218

Van Vleck. Another book in a similar vein is **Bringing Calculus to Life**, by Decker and Williams. Projects which involve the formulation of mathematical models for scientific problems and whose solution may be aided by computer software such as Maple or other technology are especially valuable for students interested in physical sciences and engineering. Maple (and Matlab) are readily available to students, but you can't assume the TAs know these technologies.

Computer software can improve teaching and learning in calculus. Our large lecture rooms and several of the smaller classrooms have equipment which can be used to advantage at many points in the 221-222-234 sequence. Room B107 has workstations which can be used by students.

Some of the ways computer technology can be used in teaching calculus amount to improved versions of pictures or calculations which the teacher has traditionally done on the board, while others simply would not have been practical without the computer. Some specific suggestions are:

(1) Computer generated "movies" can show the tangent line moving along a curve, illustrating both how the slope changes and how the line approximates the curve near the point of tangency.

(2) Software can illustrate graphically changing solutions to a differential equation as the initial conditions are changed. This can be done at an early stage, when differential equations are treated informally as solution for position in terms of velocity, or later when there is a more formal study. The graphical illustrations can be more or less detailed, a few curves through different points or complete direction fields with overlaid solutions.

(3) A simple construction in Maple will accept a function, an interval, and a number of (equal length) subintervals, and will both compute and graphically illustrate Riemann sums. Varying the choice of right, left, upper, or lower sums, and refining the partition, gives the students a much better feel for the limit underlying the integral than any static picture.

(4) Graphs of functions of two variables, produced by the computer using color and shading, are much easier for students to see as three dimensional than what is practical to draw on the board.

Each instructor needs to choose ways to use these technologies which are appropriate to his or her teaching style, but for virtually any style there will be opportunities to use the computer in genuinely helpful ways. The University has site licenses which make it possible for instructors to have copies of computer software such as Maple and Matlab. Those products (as well as several others) are also available in a reduced-price version for students who wish to have their own copies: Such software is also available for student and faculty use in many computer labs across campus. Several instructors have created or collected assignments which you can give to students, which are intended to make use of such software to go beyond what is normally learned in these courses or to help drive home points we usually teach. There are also published collections of such assignments, usable as supplementary texts and including "how-to" information on software which is available for student use, such as **Maple V Calculus**

Labs by Fattahi and **CalcLabs with Maple V** by Boggess, *et al.* A book which has some good laboratory ideas but which assumes that the students are using Mathematica, which we no longer have generally available, is **Calculus Laboratories with Mathematica** by Kerchhove and Hall. Another good reference **Calculus the Maple Way** by Robert Israel; this is written as a text supplement for students but has lots of things that make good class demos.

It is not necessary for the instructor to be able to use the software in order to give such assignments. It is definitely not necessary to use lecture time to teach computing skills.

Remark: Math 234 is particularly appropriate for assigning some "major" projects, requiring the students to extend or apply classwork. Either individually or in small groups they can be asked to work on problems which may take them several weeks, and which are to be written up in extensive detail. These projects might involve applications in areas the students care about, or extensions of material covered in class, possibly using the computer labs. The TAs can be expected to grade such projects.

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