Mathematics 475 **Fall 1999** **Wilson**

Your Name: ______________________________

Exam II 12/1/99

- Where possible please write your answers in the spaces provided right after the problems. If you need more space, either on the exam sheets or on extra paper, please make sure that you have made clear what work goes with what problem!
- There is a problem <u>on the back of this sheet.</u>
- There is scratch paper provided on the back of the last sheet.
- You may leave lengthy calculations unevaluated, if you give enough information that any one who can use a calculator with buttons for exponential functions, factorials, and combinatorial coefficients (as well as ordinary arithmetic) could carry them out.
- <u>How</u> you get an answer is at least as important as a final answer for these problems. Please show all work that could possibly help explain what you are doing, and feel free to write comments on your reasoning. (But don't feel you need to put in irrelevant filler. . . .)

Problem 1 (14 points)
Let h_n be the number of ways that a $1 \times n$ chessboard can be colored with the four colors Red, Blue, Green, and Orange, in such a way that no two adjacent squares are colored Red. Find a recurrence relation satisfied by h_n.
(You do not need to solve the recurrence or find a generating function for it. You should give some logical argument showing why your recurrence does describe h_n.)

Problem 2 (14 points)
Solve the recurrence relation $h_n = -h_{n-1} + 12h_{n-2}$ (for $n \geq 2$) with the initial values $h_0 = 1$ and $h_1 = -11$.

Problem 3 (14 points)
For a holiday gift-giving pool, each of 12 people writes his/her name on a slip of paper and puts it into a hat. The pieces of paper are then mixed up, after which each person draws out one name.

(a) In how many different ways can this happen so that no person draws his/her own name?

(b) In how many different ways can this happen so that at least one person draws his/her own name?

Problem 4 (15 points)
Solve the recurrence relation $h_n = -3h_{n-1} + 4n$ (for $n \geq 1$) with initial condition $h_0 = -\frac{1}{4}$.

Problem 5 (15 points)
Count the permutations $i_1 i_2 i_3 i_4 i_5$ of $\{1,\ 2,\ 3,\ 4,\ 5\}$ which satisfy $i_1 \notin \{2,\ 3\}$, $i_2 \notin \{2,\ 3\}$, $i_3 \notin \{4,\ 5\}$, and $i_4 \notin \{4,\ 5\}$. (There are no restrictions on i_5.)

Problem 6 (14 points)
Find a generating function $g(x)$ for the recurrence relation $h_n = -4h_{n-1} + 3h_{n-2}$ (for $n \geq 2$) with initial conditions $h_0 = 1$ and $h_1 = -1$.
(You do not need to find a series representation for $g(x)$, you do not need to decompose $g(x)$ using partial fractions, and you do not need to solve for h_n as an explicit function of n.)

Problem 7 (14 points)
Find the number of 12-combinations of the multiset

$$\mathcal{T} = \{3 \cdot a, 4 \cdot b, 5 \cdot c, 4 \cdot d\}.$$

SCRATCH PAPER