Your Name: ____

Mathematics 475 **Fall 1999** **Wilson**

Exam I 10/20/99

- Where possible please write your answers in the spaces provided right after the problems. If you need more space, either on the exam sheets or on extra paper, please make sure that you have made clear what work goes with what problem!
- There is scratch paper provided on the back of this sheet.
- There are seven problems, on six pages including this one and the scratch paper. Be sure that you have all of them.
- You may leave combinatorial coefficients, factorials, and similar numerical calculations in your answers. E.g., $3^5\binom{6}{4}$ is just as good an answer as 3645.
- How you get an answer is at least as important as a final answer for these problems. Please show all work that could possibly help explain what you are doing, and feel free to write comments on your reasoning. (But don't feel you need to put in irrelevant filler ...)

SCRATCH PAPER

Problem 1 (12 points)
How many ways are there to place 5 rooks on a 9×9 chess board so that no two attack each other, if 3 of them are blue and 2 of them are red?
(You can tell blue ones from red ones, but the red ones cannot be distinguished from each other and likewise for the blue ones.)

Problem 2 (12 points)
You are trying to organize your books. You have two shelves to put the books onto, and each shelf is able to hold 120 books. You have 90 Math books, 70 English books, and 80 Philosophy books. The Math books have to go on the top shelf. The English books have to go on the lower shelf. The Philosophy books can go on either shelf. How many ways are there to arrange your books so that these criteria are met?

(For this problem: You can't tell two books apart except for what subject they refer to, i.e. any two Math books are the same as any other two and the order they are in does not matter. But the order in which the subjects appear on a shelf does matter. For example, if there were five Math and Philosophy books on a shelf in the order MMPMP, that would be different from MPMMP.)

Problem 3 (18 points)
You eat at the same place for supper every evening, and choose randomly from their menu which has five different items.

(a)) How many times must you dine there before you know you must have had at least one item ten times?

(b) One of the items on the menu is pizza. Is there a number of times you can dine at this place which would guarantee that you must have chosen pizza ten times? If not, why not? If so, what is the number?

(c) Suppose you eat at this place for 50 days. How many different combinations of meals can you have? (E.g., one combination is menu item 1 for 10 meals, item 2 for 20 meals, item 3 for 20 meals, and no meals that were item 4 or 5. Order does not matter.)

Problem 4 (15 points)
In a group of 50 people, each has an even number of acquaintances in the group. (We use the word "acquaintance" here such that nobody is acquainted is with him/herself. The "even number" may be zero.)
Prove that there is some collection of at least three people in this group who all have the same number of acquaintances.

Problem 5 (14 points)
In the expansion of $(2x-4y)^{20}$:

(a) What is the coefficient of x^5y^{15}?

(b) What is the coefficient of x^6y^{14}?

Problem 6 (15 points)
Evaluate the sum

$$1-\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}-\frac{1}{4}\binom{n}{3}+\cdots+(-1)^n\frac{1}{n+1}\binom{n}{n}.$$

Hint: Use a formula for $(1-x)^n$, integrate, choose a good value for x ....

Problem 7 (14 points)
Let $X = \{A, B, C, D\}$.

(a) How many 2-combinations are there from the set X? List all of them.

(b) How many 2-permutations are there from the set X? List all of them.