Your Name: ______________________________

Mathematics 443 (Wilson) | Exam 2 | April 4, 1997

You have 50 minutes for this exam. Write your answers to the five problems in the spaces provided. There is some extra space below and at the end of the exam. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem. There are problems on the backs of sheets of paper: Be sure you see all five problems!

Problem 1

Let $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & -2 \\ 2 & 5 & 6 \end{bmatrix}$.

Find the adjoint of A **and the determinant of** A**, and check that** A **adj**(A) $=$**det**(A) I**.**

Problem 2

Let V **be the space** P^3 **of all polynomials of degree less than 3, with real coefficients and using the reals as scalars. For each of the following tell whether it is a subspace of** V **and give reasons. (Let a polynomial in** V **be represented as** $a_2x^2 + a_1x + a_0$**.)**

(a) The set of polynomials such that $a_2 + a_1 = a_0$**.**

(b) The set of polynomials such that $a_2 + a_1 = a_0 + 2$**.**

(c) The set of polynomials such that $a_2 = 0$**.**

Problem 3

Consider the four 3-element column vectors of real numbers $v_1 = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$, $v_3 = \begin{pmatrix} -3 \\ -1 \\ 0 \end{pmatrix}$, **and** $v_4 = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$.

Let V **be the subspace of** $\Re^3$ **spanned by** $\{v_1, v_2, v_3, v_4\}$.
Find a basis for V. **What is the dimension of** V**?**

Problem 4

Let B_1 **and** B_2 **be the ordered bases** $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$ **and** $\left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$ **for** $\Re^3$ **respectively.**

(a) Find the coordinates of the vector $\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$ **with respect to each basis.**

(b) Find the 3×3 **matrix** M **such that, for any** $v \in \Re^3$, $v_{B_1} = M v_{B_2}$. **(**v_B **denotes the coordinates of** v **with respect to** B**.)**

Problem 5

Let $A = \begin{bmatrix} 1 & 1 & 2 & 0 \\ -1 & -1 & -2 & 1 \\ -1 & -1 & -2 & -1 \end{bmatrix}$.

(a) Find a basis for the row space $R(A)$**.**

(b) Find a basis for the column space $C(A)$**.**