Your Name: ______________________________

Mathematics 431 (Wilson) Final Exam August 7, 1997

- You have 75 minutes for this exam.
- At the end of the exam you will find a table for the normal random variable with mean 0 and standard deviation 1.
- Be sure to show your reasoning: A correct numeric answer without supporting work may receive reduced or zero credit.
- Unless instructed otherwise you should carry calculations out to the point where all that is left is simple arithmetic and/or the specific values of universal constants such as e or π.
- Write your answers to the 9 problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.
- There are problems on the backs of sheets of paper: Be sure you see all 9 problems!
- You may use the space at the bottom of this page, space after the last problem, and space on the sheet with the table of $\Phi(x)$, for scratch paper. I won't look there for answers unless you specifically direct me to do so at the place where the problem is printed.

Problem 1

An assembly line makes millions of toasters. It is known that one percent of the toasters coming from this line are defective. One thousand toasters are selected at random for testing. What is the probability that sixteen or more of the tested toasters will be defective? (Find a number, don't just set up a calculation.)

Problem 2

A class contains 3 math majors, 2 engineering majors, and 12 other students. Four students are selected at random. X is the number of math majors among the four students who are selected, and Y is the number of engineering majors in the selected four. Fill in the table below with the joint probability mass function and the marginal mass functions for the joint random variables X and Y:

i \ j	0	1	2	$P\{X = i\}$
0				
1				
2				
3				
$P\{Y = j\}$				

Problem 3

Suppose X is a continuous random variable whose probability density function is given by

$$f(x) = \begin{cases} c(x+2) & \text{if } -2 \leq x < 0 \\ c(2-x) & \text{if } 0 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) What must be the value of c?

(b) Find the cumulative distribution function $F(a)$ for X.

(c) What is $P\{X > -1\}$?

Problem 4

Suppose the joint distribution function for X and Y is given by $f(x, y) = \frac{1}{9}(2x + 4y + xy)$ for $0 \leq x \leq 2$ and $0 \leq y \leq 1$, and $f(x, y) = 0$ for all other values of x and y.
Find $P\{X > 1 | Y = y\}$.

Problem 5

There are five boxes. Each contains some pennies and some dimes: In the first box there are 2 pennies and 6 dimes. In the second box there are 7 pennies and 3 dimes. In the third box there are 3 pennies and 2 dimes. In the fourth box there are 3 pennies and 3 dimes. In the fifth box there are 1 penny and three dimes. From each box we pick one coin at random, getting a total of five coins. What is the expected number of pennies among our five coins?

Problem 6

A dairy packages milk in bottles which are supposed to contain 1 kilogram (1000 grams). From past records it is believed that the amounts of milk in the bottles are a collection of independent and identically distributed random variables with a common variance of 100 grams and some common mean μ that the dairy puts in a bottle on the average.

Officials suspect that the dairy is underfilling the bottles. They are going to weigh the contents of a random sample of n bottles, and use the average of those weights as their estimate of how much the dairy puts in a typical bottle. How many bottles (n) must the officials weigh in order to have at least 0.95 probability that their estimate is within 5 grams of μ? (You can interpret that as meaning $-5 \leq \overline{X} - \mu \leq 5$, in case you are wondering about whether to use $\leq$ or $<$.)

Problem 7

(a) Fish taken from a certain lake have an average length of 10 inches. If a fish from this lake is measured, find an upper bound for the probability that the fish is at least 18 inches long.

(b) A more careful study determines that the lengths of the fish in this lake have a variance of 6 inches. Use this additional information to improve your bound from (a), *i.e.* to find a smaller bound for the probability that a randomly selected fish has length at least 18 inches.

Problem 8

We believe that 70% of the people in Wisconsin prefer butter over margarine. If we take a random sample of 1000 people from Wisconsin, what is the probability that at least 80% of the people in our sample prefer butter?

Problem 9

A die is rolled twice. X is the number of times an even number appears, and Y is the number of times we get a 4, 5, or 6. (*E.g.* if we get 5 on the first roll and 4 on the second, $X = 1$ and $Y = 2$.) Find the covariance $Cov(X, Y)$ and the correlation $\rho(X, Y)$.

Area $\Phi(x)$ under the Standard Normal Curve to the Left of x:

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998