Your Name: _____

Mathematics 431 (Wilson) Final Exam August 8, 1996

- You have 75 minutes for this exam.
- At the end of the exam you will find a table for the normal random variable with mean 0 and standard deviation 1.
- Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.
- There are problems on the backs of sheets of paper: Be sure you see all eight problems!
- You may use the space at the bottom of this page and the space below the table of $\Phi(x)$ for scratch paper. If you need more scratch paper there is some on the table at the front of the room.

Problem 1

A box contains 10 red and 5 black balls. A ball is selected at random from the box. If the selected ball is red it is put back into the box, but if it is black it is put back and two additional black balls are also put into the box. A second ball is selected at random from whatever balls are now in the box.

(a) Describe a sample space for this experiment. List all of the outcomes.

(b) What is the probability that the second ball selected is red?

Problem 2

A student is taking a multiple choice exam. Each question has five possible answers, one right and four wrong. If the student knows the answer he selects the correct answer. If the student does not know the answer he guesses, picking randomly one of the five choices. The student knows the answers to 70% of the questions.

(a) What is the probability that on a given question the student gets the correct answer somehow?

(b) If the student gets the correct answer to a question, what is the probability that he knows the answer to that question?

Problem 3

A machine produces screws, 1% of which are defective. Whether one screw is or is not defective has no relation to whether any other screw is defective or is not. Find the probability that a box of 200 screws contains NO defective screws:

(a) Directly, calculating from a binomial distribution.

(b) Approximately, using a Poisson approximation.

Problem 4

An urn contains 2 red, 1 black, and 3 white balls. From the urn, 3 balls are selected at random. Let X denote the number of red balls which are selected and Y denote the number of black balls which are selected. Fill in the table below with the joint and marginal probability mass functions.

i \ j	0	1	$P\{X = i\}$
0			
1			
2			
$P\{Y = j\}$			

Problem 5

Suppose that the weight of a person selected at random from some population is normally distributed with parameters μ and σ. Suppose also that $P\{X \leq 160\} = \frac{1}{2}$ and $P\{X \leq 140\} = \frac{1}{4}$.

(a) Find μ and σ.

(b) What is the probability that a randomly selected person from this population weighs at least 200 pounds?

(c) Of all the people in this population who weigh at least 200 pounds, what percentage will weigh over 220 pounds?

Problem 6

Suppose an urn contains 3 balls, which are labeled 1, 2, and 3. Two balls are selected without replacement from the urn. Let X be the number on the first ball selected and let Y be the number on the second ball selected. Compute the covariance $\mathrm{Cov}(X, Y)$ and the correlation $\rho(X, Y)$.

Problem 7

Candidates A and B are running for office and we know that 55% of the voters prefer candidate B. If we ask a random sample of 100 voters for their preferences, what is the probability that at least one half of those whom we ask will prefer candidate A?

Problem 8

Let X be a discrete random variable whose probability mass function is

$$p(x) = \begin{cases} \frac{1}{18} & \text{if } x = 1 \text{ or } x = 3 \\ \\ \frac{16}{18} & \text{if } x = 2 \end{cases}$$

We know by Chebyshev's inequality that

$$P\{|X - \mu| \geq k\} \leq \frac{\sigma^2}{k^2}$$

Find a value of k such that for this random variable X and your choice of k,

$$P\{|X - \mu| \geq k\} = \frac{\sigma^2}{k^2}$$

which shows that Chebyshev's inequality cannot be made stronger and still be true in general.

Area $\Phi(x)$ under the Standard Normal Curve to the Left of x:

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998