Your Name: _____

Mathematics 431 (Wilson)   Exam 2   July 17, 1997

1. You have 75 minutes for this exam.

2. Write your answers to the seven problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

3. There is scratch paper at the end of the exam. I will not look there for answers unless you specifically point me there as in (2) above.

4. There are problems on the backs of sheets of paper: Be sure you see all seven problems!

5. You are welcome to leave symbols like $\binom{n}{r}$ and numbers such as e^{-2} in your answers.

Problem 1

A box of candy has two kinds of candy, hard and soft, and two flavors, chocolate and lemon. There are 15 hard chocolate candies, 10 soft chocolate candies, and 12 soft lemon candies. A candy is going to be chosen at random from the box. How many hard lemon candies must there be in order for the events {the candy chosen is chocolate} and {the candy chosen is soft} to be independent?

Problem 2

A random variable X takes on (only) the values 1, 3, 5, and 8, with probabilities $p(1) = \frac{1}{2}$, $p(3) = \frac{1}{8}$, $p(5) = \frac{1}{8}$, and $p(8) = \frac{1}{4}$.

(a) Sketch a graph of the cumulative distribution function F for X.

(b) What is the expected value $E[X]$?

(c) What is the variance $V[X]$?

Problem 3

There are three dice in a box. One is a standard die with faces numbered 1, 2, 3, 4, 5, and 6. Another die has its faces numbered 1, 1, 2, 2, 3, and 3. The remaining die has the number 3 on all six of its faces. (Each die is properly balanced, so that each of its six faces is equally likely to come out on top.) One of the dice is picked at random from the box and rolled. The number on the top face of that die is observed.

(a) What is the probability that the number observed is 3?

(b) If the number observed is 3, what is the probability that the die which was rolled was the one with 3 on all of its faces?

Problem 4

An assembly line is producing toasters. It is known that 0.3% of the toasters produced are defective. One thousand toasters are selected at random for testing.

(a) How many of the tested toasters do you expect to be defective?

(b) What is the variance of the number of defective toasters in the tested group?

(c) Set up a calculation which would give the probability that five or more tested toasters are defective, using the binomial random variable.

(d) Set up a calculation which would give the probability that five or more tested toasters are defective, using the Poisson random variable.

Problem 5

A computer network has to tolerate errors which occur at random times. They are observed at an average rate of one every minute. What is the probability that 4 or more errors occur in a 2 minute interval?

Problem 6

A brand of soda has colored dots hidden under the bottle caps. Nine out of ten dots are red, and one out of ten is green. If you collect 3 green dots you win a prize. How many sodas do you expect to buy before winning a prize?

Problem 7

Two dice are rolled: One is green and the other is blue. A random variable is computed as

$$X = (\text{the number on the green die}) - (\text{the number on the blue die})$$

(a) What values can X take on?

(b) What is the probability associated with each value in (a)?

(c) What is the expected value $E[X]$?

(d) What is the standard deviation $\sigma[X]$?

Scratch Paper