Your Name: _____

Mathematics 431 (Wilson) Exam 2 July 23, 1996

- **You have 75 minutes for this exam.**
- **Write your answers to the seven problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.**
- **There is scratch paper at the end of the exam. I will not look there for answers unless you specifically point me there with a note where the answer belongs.**
- **There are problems on the backs of sheets of paper: Be sure you see all seven problems!**
- **You are welcome to leave symbols like $\binom{n}{r}$ in your answers.**

Problem 1

A fair coin (equally likely heads and tails) is flipped repeatedly until the first time that heads occurs. The number of flips is recorded as a random variable. (*E.g.* for the sequence TTH the variable gives 3.)
(a) What are the values taken on by this random variable?
(b) What is the probability mass function $p(i)$ for this random variable?
(c) What is the cumulative distribution function $F(x)$ for this random variable?
(d) Set up a calculation for the expected value $\mathbf{E}[X]$ for this random variable: You do not need to carry out the calculation.

Problem 2

The cumulative distribution function $F(x)$ for a random variable X is given by

$$F(x) = \begin{cases} 0 & \textbf{for} \quad x < -2 \\ 1/10 & \textbf{for} \quad -2 \le x < -1 \\ 1/4 & \textbf{for} \quad -1 \le x < 0 \\ 0.5 + x/8 & \textbf{for} \quad 0 \le x < 1 \\ 3/4 & \textbf{for} \quad 1 \le x < 2 \\ 1 & \textbf{for} \quad 2 \le x \end{cases}$$

(a) What is $P\{X = 1\}$?
(b) What is $P\{X > \frac{1}{2}\}$?
(c) What is $P\{-1 < X < 3\}$?

Problem 3

We roll a fair die 2 times and add the results, getting a number $X = 2 \ldots 12$. Find the expected value $\mathbf{E}[X]$ and the variance $\mathbf{Var}(X)$.

Problem 4

Let X be a binomial random variable with parameters $n = 5$ and $p = 0.3$.
(a) What is the expected value $\mathbf{E}[X]$?
(b) What is the variance $\mathbf{Var}(X)$?
(c) What is the probability $P\{X = 4\}$?
(d) What is the probability $P\{X \le 4\}$?

Problem 5

There are lots of cars in this country. Every minute, on the average, 7 of them develop transmission trouble. Approximately what is the probability that in a given minute 3 or fewer cars develop transmission trouble?

Problem 6

Let X be a Poisson random variable with parameter $\lambda = 0.4$.
(a) What is the probability $P\{X = 3\}$?
(b) What is the probability $P\{X \geq 4\}$?
(c) What are the expected value $\mathbf{E}[X]$ and the variance $\mathbf{Var}(X)$?

Problem 7

The University telephone exchange notes that on the average three calls are made each second. What is the probability that five seconds go by with no calls?

Scratch Paper