Mathematics 431 (Wilson)   Exam 1   July 1, 1997

1. You have 75 minutes for this exam.

2. Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

3. There is scratch paper at the end of the exam. I will not look there for answers unless you specifically point me there as in (2) above.

4. There are problems on the backs of sheets of paper: Be sure you see all eight problems!

5. When I refer to flipping a coin I mean that it produces two possible outcomes, labeled H and T: For a fair coin those outcomes are equally likely.

6. You are welcome to leave symbols like $\binom{n}{r}$ in your answers.

Problem 1

A graduation requirement for your major contains two lists of courses: List A contains 7 courses and list B has 8 courses. None of these courses has any of the others as prerequisites and it doesn't make any difference what order you take them in.

(a) How many ways can you meet the requirement if it says you must take (any) 12 out of the courses on lists A and B?

(b) How many ways can you meet the requirement if it says you must take 6 courses from list A and 6 courses from list B?

Problem 2

A small town is analyzing how people commute. Everyone who commutes uses either a bicycle or a car. It is found that 30% of the population commute only by bicycle and that 57% commute by bicycle at least some of the time, while 3% never commute at all.

(a) What percentage of the population commute by car at least some of the time?

(b) If a person who is picked randomly from this town is found to commute by car at least some times, what is the probability that he or she also bicycles some of the time?

Problem 3

An urn starts out with 5 white and 4 black balls. We draw a ball at random out of the urn, write down what color it is, and then put that ball back and also add an additional ball of the same color. We then repeat this operation two more times. (For example if a black ball is drawn the first time, then before the next draw the urn will contain 5 white and 6 black balls.)

What is the probability that the three balls drawn alternate in color, *i.e.* are either black-white-black or white-black-white?

Problem 4

Urn A contains 3 red balls and 3 white balls. Urn B contains 2 red balls and 4 white balls. You perform the following experiment: You first flip a fair coin. If it produces Heads then you draw a ball from urn A and note its color, while if the coin produces Tails you draw the ball from urn B instead.

(a) Describe a sample space for this experiment. You may represent the outcomes with words or symbols but be sure to tell what your symbols mean.

(b) How many outcomes are there in your sample space?

(c) Are the outcomes in your sample space equally likely? Why or why not?

Problem 5

You are at an amusement park where there are five different rides. Each ride costs one dollar, and you have ten dollars.

(a) In how many ways can you choose which rides to go on, if the order in which you take the rides does not make any difference?

(b) In how many ways can you choose which rides to go on, if you want to make sure you try each ride at least once? (The order still does not make a difference.)

Problem 6

Urn A contains 2 red and 4 black balls.
Urn B contains 4 red and 5 black balls.
Urn C contains 7 red and 3 black balls.
A ball is chosen (at random) from each urn.

What is the probability that the ball chosen from urn B was red if it is known that some two of the chosen balls were the same color as each other? (Those two might or might not be the same color as the ball from urn B.)

Problem 7

Urn A contains 5 red and 2 black balls, while urn B contains 4 red and 1 black balls.
A ball is randomly chosen from each urn.

What is the probability that the two balls will be the same color?

Problem 8

Assume that the initial letter of a person's last name is equally likely to be any of the
26 letters A-Z of the English alphabet. If there are N people in a room, what is the
probability that at least two of them have the same initial? (Your answer should show
how the probability changes with N.)

Scratch Paper