

Your Name: _____

Mathematics 431 (Wilson)

Exam 1

July 2, 1996

- You have 75 minutes for this exam.
- Write your answers to the nine problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.
- There is scratch paper at the end of the exam. I will not look there for answers unless you specifically point me there with a note where the answer belongs.
- There are problems on the backs of sheets of paper: Be sure you see all nine problems!
- Where I refer to dice, each die is assumed to be a (six-sided) cube with the numbers 1, 2, ..., 6 on the faces. An outcome of rolling a die is one of the six numbers 1...6, all equally likely.
- When I refer to flipping a coin I mean that it produces two possible outcomes, labeled H and T: For a fair coin those outcomes are equally likely.
- You are welcome to leave symbols like $\binom{n}{r}$ in your answers.

Problem 1

Urn A contains 5 red and 6 black balls, while urn B contains 7 red and 6 black balls. A ball is randomly chosen from each urn.

What is the probability that the two balls will be different in color?

Problem 2

You are given 30 identical marbles. You have to put them into 6 boxes: Each box is big enough that it could hold all the marbles.

(a) If you can put any number of marbles into each box, but must use all the marbles, how many ways can you accomplish this?

(b) If you cannot leave any box empty, and still must use all the marbles, how many ways can the marbles be put into the boxes?

Problem 3

An urn starts out with 7 white and 9 black balls. We draw a ball at random out of the urn, write down what color it is, and then put that ball back and also add an additional ball of the opposite color. We then repeat this operation two more times. (For example if a white ball is drawn the first time, then before the next draw the urn will contain 7 white and 10 black balls.)

What is the probability that the three balls drawn alternate in color, *i.e.* are either black-white-black or white-black-white?

Problem 4

On some other test you have to answer 15 out of 20 questions.

(a) If you are free to choose any 5 questions to skip, how many different ways can you choose which questions to answer?

(b) If you have to answer 7 out of the first 10 questions and 8 out of the last 10 questions, how many ways can you choose which questions to answer?

Problem 5

Assume that there are N people in a class and that each of them has a (random, 9-digit) Social Security number. Each person is asked to tell the last two digits of his or her Social Security number.

What is the probability that some two people give the same answer? (Your answer should show how the probability changes with N .)

Problem 6

Urn A contains 3 red and 6 black balls.

Urn B contains 4 red and 7 black balls.

Urn C contains 7 red and 7 black balls.

A ball is chosen (at random) from each urn.

What is the probability that the ball chosen from urn B was black if it is known that two of the chosen balls were red?

Problem 7

A survey asks participants whether they watch news on TV or read a newspaper. The results show that 40% read a newspaper and 70% watch TV, and 20% do both.

(a) How many (as a percentage) neither watch TV nor read a newspaper?

(b) If a participant is picked at random and turns out to be a newspaper reader, what is the probability that he is also a TV watcher?

Problem 8

You perform the following experiment: First you flip a (fair) coin. If the coin produces H, you then roll a die and note what number it produces. If instead the coin produces T, you then flip the coin again and write down whether it produces H or T this time.

(a) Describe a sample space representing the outcomes of this experiment: You may either write out the description in words or assign symbols to the outcomes.

(b) How many outcomes are there?

(c) Are all outcomes equally likely? Why or why not?

Problem 9

Customers can buy cake and/or ice cream at a restaurant. There are two flavors of ice cream, vanilla and strawberry, and a customer can buy either flavor by itself, or both, or neither, and can buy cake or not buy cake regardless of the ice cream decision. Sales records show that:

33% of customers buy strawberry ice cream; 31% of customers buy vanilla ice cream; 38% of customers buy cake; 6% of customers buy both kinds of ice cream; 11% of customers buy cake and strawberry ice cream; 19% of customers buy cake and vanilla ice cream; 4% of customers buy cake and both kinds of icecream.

(a) What is the probability that a customer will buy exactly two items, either the two kinds of ice cream or one kind of ice cream with cake?

(b) What is the probability that a customer buys some cake or ice cream, no matter what combination?

Scratch Paper