Problem 1:
(a) The possible values for the random variable are $1,\ 2,\ 3, \ldots$, which you could describe by some phrase such as the positive whole numbers or the whole numbers greater than zero.
(b) One and only one outcome leads to the value i, where i is some positive whole number, namely $i-1$ tails followed by 1 head. That outcome has probability $(1/2)^{i-1}(1/2) = (1/2)^i$ which can also be written as $1/(2^i)$.
(c) For $i = 1,\ 2,\ 3, \ldots$,

$$F(x) = \sum_{i=1}^{x} p(i) = \sum_{i=1}^{x} \left(\frac{1}{2}\right)^i = 1 - \frac{1}{2^i}$$

(d)

$$E[x] = \sum_{i=1}^{\infty} \frac{i}{2^i}$$

which works out to 2, but you did not have to sum the series.

Problem 2:
(a)

$$P\{x=1\} = F(1) - \lim_{x\to 1^-} F(x) = F(1) - \lim_{x\to 1^-}\left(\frac{1}{2}+\frac{x}{8}\right) = \frac{3}{4} - \frac{5}{8} = \frac{1}{8}$$

(b) $P\{X > \frac{1}{2}\} = 1 - F(\frac{1}{2})$ and $F(\frac{1}{2}) = \frac{9}{16}$. Thus $P\{X > \frac{1}{2}\} = \frac{7}{16}$.
(c)

$$P\{-1 < x < 3\} = \lim_{x\to 3^-} F(x) - F(1).$$

Since F is 1 for any $x \geq 2$, $\lim_{x\to 3^-} F(x) = 1$. Thus the answer is $1 - \frac{1}{4} = \frac{3}{4}$.

Problem 3

$$E[X] = 2\cdot\frac{1}{36} + 3\cdot\frac{2}{36} + 4\cdot\frac{3}{36} + 5\cdot\frac{4}{36} + 6\cdot\frac{5}{36} + 7\cdot\frac{6}{36} + 8\cdot\frac{5}{36} + 9\cdot\frac{4}{36}$$

$$+10\cdot\frac{3}{36} + 11\cdot\frac{2}{36} + 12\cdot\frac{1}{36} = 7$$

$$E[X^2] = 4\cdot\frac{1}{36} + 9\cdot\frac{2}{36} + 16\cdot\frac{3}{36} + 25\cdot\frac{4}{36} + 36\cdot\frac{5}{36} + 49\cdot\frac{6}{36} + 64\cdot\frac{5}{36} + 81\cdot\frac{4}{36}$$

$$+100\cdot\frac{3}{36} + 121\cdot\frac{2}{36} + 144\cdot\frac{1}{36} = \frac{1974}{36}$$

$$Var[X] = E[X^2] - (E[X])^2 = \frac{1974}{36} - 49 = 5\frac{5}{6} \approx 5.833$$

Problem 4

(a) For a binomial random variable, $E[X] \;=\; np \;=\; 5 \times 0.3 \;=\; 1.5.$

(b) $\mathrm{Var}[X] \;=\; np(1-p) \;=\; 5 \times 0.3 \times 0.7 \;=\; 1.05.$

(c)

$$P\{X = 4\} = \binom{5}{4}(0.3)^4(0.7)^1 = 5 \times 0.0081 \times 0.7 = 0.2835$$

(d)

$$P\{X \leq 4\} = 1 - P\{X = 5\} = 1 - \binom{5}{5}(0.3)^5(0.7)^0 = 1 - (0.3)^5 = 1 - 0.00243 = 0.99757$$

Problem 5

Since the number of cars N is large and the probability of failure in a given minute is $p = 7/N$, we can approximate the failure rate with a Poisson random variable with $\lambda = pN = 7$. That has $p(i) = e^{-\lambda}\lambda^i/i!$. Then $P\{X \leq 3\} = p(0) + p(1) + p(2) + p(3) = e^{-7}(1 + 7 + \frac{49}{2} + \frac{343}{6}) \approx 0.0818$.

Problem 6

(a)

$$P\{X = 3\} = e^{-\lambda}\frac{\lambda^3}{3!} = e^{-0.4}\frac{0.4^3}{6} \approx 0.00715$$

(b)

$$P\{X \geq 4\} = 1 - P\{X \leq 3\} = 1 - e^{-\lambda}\left(1 + \lambda + \frac{\lambda^2}{2} + \frac{\lambda^3}{6}\right)$$

$$= 1 - e^{-0.4}\left(1 + 0.4 + \frac{0.16}{2} + \frac{0.064}{6}\right) \approx 0.000776$$

(c) For a Poisson variable the Expectation and Variance are each equal to λ, 0.4.

Problem 7

If the average rate is three calls each second, the average number of calls in 5 seconds is 15. We use a Poisson random variable X with $\lambda = 15$ and find $P\{X = 0\} = p(0) = e^{-15} \approx 3.06 \times 10^{-7} = 0.000000306$.