Your Name: ______________________________

Mathematics 340, Spring 2007 **Lecture 1 (Wilson)**

Second Midterm Exam March 29, 2007

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

There is a problem on the back of this sheet: Be sure not to skip over it by accident!

There is scratch paper at the end of this exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought with you, as described in email to the class.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. If you are citing a theorem or other result from the book, you do not need to give theorem number or page, but you should make clear exactly what the cited result says. Don't invent new things in the book that make a problem trivial.

If a problem refers to a vector, like $\vec{v}$ or $\vec{v}_1$, it can be in any vector space unless there is a particular space specified. In any one problem you may assume all vectors come from the same space unless specified otherwise.

The symbol $\mathbb{R}$ is used to denote the set of all real numbers. $\mathbb{R}^n$ and $\mathbb{R}_n$ denote, respectively, the sets (vector spaces) of all n-element column vectors and of row vectors.

Problem	Points	Score
1	13	
2	13	
3	12	
4	12	
5	13	
6	12	
7	12	
8	13	
TOTAL	100	

Problem 1 (13 points)

Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

(a) Show that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a basis for $\mathbb{R}^3$.

(b) Express $\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$ as a linear combination of $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$.

Problem 2 (13 points)
Show that if $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent and $\vec{v}_3$ does not belong to the span of $\{\vec{v}_1, \vec{v}_2\}$ then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent.

Problem 3 (12 points)

Find a basis for the subspace W of $\mathbb{R}^3$ spanned by $\left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 11 \\ 10 \\ 7 \end{bmatrix}, \begin{bmatrix} 7 \\ 6 \\ 4 \end{bmatrix} \right\}$.

What is the dimension of W?

Hint: The RREF of $\begin{bmatrix} 1 & 3 & 11 & 7 \\ 2 & 2 & 10 & 6 \\ 2 & 1 & 7 & 4 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Problem 4 (12 points)

(a) Find a basis for, and the dimension of, the solution space of the system

$$\begin{aligned} x_1 + 2x_2 - x_3 + 3x_4 &= 0 \\ 2x_1 + 2x_2 - x_3 + 2x_4 &= 0 \\ x_1 + 3x_3 + 3x_4 &= 0 \end{aligned}$$

Hint: The RREF of $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 2 & -1 & 2 \\ 1 & 0 & 3 & 3 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & \frac{8}{3} \\ 0 & 0 & 1 & \frac{4}{3} \end{bmatrix}$.

(b) Let $A = \begin{bmatrix} 1 & -2 & 7 & 0 \\ 1 & -1 & 4 & 0 \\ 3 & 2 & -3 & 5 \\ 2 & 1 & -1 & 3 \end{bmatrix}$

Find a basis for the column space of A, consisting of some columns of A.

Hint: The RREF of A is $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Problem 5 (13 points)

Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 9 & -1 \\ -3 & 8 & 3 \\ -2 & 3 & 2 \end{bmatrix}$

Find a basis for the row space of A, consisting of vectors that are rows of A.

Problem 6 (12 points)

Let A be a 7×3 matrix whose rank is 3.

(a) Are the rows of A linearly independent or linearly dependent? Justify your answer.

(b) Are the columns of A linearly independent or linearly dependent? Justify your answer.

Problem 7 (12 points)

Define a function L from $\mathbb{R}^3$ to $\mathbb{R}^3$ by $L\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}\right) = \begin{bmatrix} u_1 + 4u_2 \\ -u_3 \\ u_2 + u_3 \end{bmatrix}$.

(a) Show that L is a linear transformation.

(b) Find the standard matrix representing L, i.e. a matrix A such that $L\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}\right) = A\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ for any vector $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ in $\mathbb{R}^3$.

Problem 8 (13 points)
Let $L : \mathbb{R}_4 \longrightarrow \mathbb{R}_2$ be the linear transformation defined by $L([u_1\ u_2\ u_3\ u_4]) = [u_1 + u_3,\ u_2 + u_4]$.

(a) Is $[2\ \ 3\ \ -2\ \ 3]$ in the kernel of L?

(b) Is $[4\ \ -2\ \ -4\ \ -2]$ in the kernel of L?

(c) Find a basis for the kernel of L.

Scratch Paper

(not a command!)