Mathematics 234, Lecture 3 (Wilson)

Your Name: ____________________________________

Circle your TA's name:

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Exam I 10/15/98

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using π, $\sqrt{3}$, and similar numbers) rather than using decimal approximations.

There is scratch paper on the back of this sheet. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on one sheet of paper (regular notebook or typing size) as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	10	
2	12	
3	13	
4	12	
5	14	
6	12	
7	15	
8	12	
TOTAL	100	

SCRATCH PAPER

Problem 1 (10 points)
For the function given by $f(x, y, z) = x\,y^2 - e^{xyz}$, find all second partial derivatives.

Problem 2 (12 points)
Let $f(x, y) = x^2 - 3xy + 5$.

(a) Find an equation for the tangent plane to the graph of $f(x, y)$ at the point where $x = 2$ and $y = 1$.

(b) Use differentials to find an approximate value of $f(1.9, 1.01)$.

Problem 3 (13 points)
Find the length of the curve

$$\vec{r}(t) = \frac{2\sqrt{2}}{3}\, t^{\frac{3}{2}}\, \vec{i} + t\ \cos(t)\ \vec{j} + t\ \sin(t)\ \vec{k}, \qquad 0 \le t \le \pi.$$

Problem 4 (12 points)
Let $f(x,y,z) = \frac{x}{y} - y\,z - 4\sqrt{x\,y\,z}$ and let P be the point $(4,1,1)$.

(a) Find a unit vector $\vec{u}$ in the direction in which $D_{\vec{u}}f$ is largest at P.

(b) What is $D_{\vec{u}}f$ at P? (answer is a number!)

(c) What is the derivative of f at P, in the direction of the vector $\vec{v} = -2\vec{i} + 2\vec{j} - \vec{k}$?

(d) Find an equation for the tangent plane to the level surface of f through P.

Problem 5 (14 points)
For the curve

$$\vec{r}(t) = 6\sin(2t)\,\vec{i} + 6\cos(2t)\,\vec{j} + 5\,t\,\vec{k}$$

find the unit tangent vector $\vec{T}$, the principal unit normal vector $\vec{N}$, the unit binormal vector $\vec{B}$, and the curvature κ at the point where $t = \frac{\pi}{2}$.

Problem 6 (12 points)
If a particle moves with acceleration $\vec{a}(t) = -\vec{i} + \vec{j} - 2\vec{k}$, starting at the point $(10, 10, 10)$ with velocity $\vec{v}(0) = -\vec{j}$ when $t = 0$, find its position vector $\vec{r}(t)$ as a function of t.

Problem 7 (15 points)
Let $f(x,y) = x^3 - 9x^2 + 24x + y^3 + 3y^2 - 2$.

(a) Find all local maxima, local minima, and saddle points of f. Be sure to tell which points are maxima, which are minima, and which are saddle points.

(b) Find the absolute maximum and minimum values of f on the closed rectangle described by the inequalities $-1 \le x \le 5$ and $-1 \le y \le 1$.

Problem 8 (12 points)
The equation $x\,y\,z + x\,e^y - x\cos(z) = 0$ defines z as a function of x and y near the point $(1,0,0)$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(1,0,0)$.