Your Name: ____

Circle your TA's name:

James Hunter **Deigo Galindo** **Nansen Petrosyan** **Boian Popunkiov**

Mathematics 234, Fall 2005 **Lecture 2 (Wilson)**

Final Exam **December 20, 2005**

Write your answers to the ten problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, draw a box around your final answer to each problem or part of a problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is a problem on the back of this sheet: Be sure not to skip over it by accident!

There is scratch paper at the end of the exam.

You may refer to notes you have brought on index cards or notebook paper, as announced in class and on the class website.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	20	
TOTAL	200	

Problem 1 (20 points)
One of the following two vector fields is conservative and the other is not:

$$\overrightarrow{F_1}(x,y) = (3 + 2xy)\vec{\imath} + (x^2 - 3y^2)\vec{\jmath}$$

$$\overrightarrow{F_2}(x,y) = (x - y)\vec{\imath} + (x - 2)\vec{\jmath}$$

(a) Which vector field is conservative? Which one is not conservative?

Show work that leads to your conclusion: You should actually show that one is conservative and show that the other is not, directly. Don't just show one is conservative, or one is not, and use the fact that there is one of each to decide about the other!

(b) For the vector field $\overrightarrow{F}$ that you found to be conservative, use the fact that it is conservative to evaluate

$$\int_C \overrightarrow{F} \cdot d\vec{r}$$

where C is the curve $\vec{r}(t) = e^t \sin(t)\vec{\imath} + e^t \cos(t)\vec{\jmath}$ with $0 \le t \le \pi$.

Problem 2 (20 points)

For $f(x, y) = e^{2x-2} \cos(x - \frac{y}{2})$:

(a) Find an equation for the tangent plane to the graph of $z = f(x, y)$ at the point where $x = 1$ and $y = 2$.

(b) Find an approximate value for $f(1.1, 1.9)$.
Your answer must visibly use calculus! Don't just use a calculator to work out the function at that point.

Problem 3 (20 points)
Use an integral to find the volume of the region that is
(a) under the paraboloid $z = x^2 + y^2$,
(b) inside the cylinder $x^2 + y^2 = 2x$, and
(c) above the plane $z = 0$.

(You must both set up the integral and evaluate it. It may be easier to do the evaluation using polar coordinates.)

Problem 4 (20 points)
Set up as an iterated integral but do not evaluate

$$\iiint\limits_S (xy - y\sin(z))\, V$$

where S is the region in space that is
(a) under the parabolic sheet $z = 4 - x^2$,
(b) above the coordinate plane $z = 0$,
(c) where $y \geq 0$, and
(d) above the plane $z = y - 2$.

Problem 5 (20 points)
Find all local maximum points, local minimum points, and saddle points for

$$f(x, y) = x^4 - 4xy + y^4 + 2.$$

Be sure to identify each point that you list as to maximum, minimum, or saddle point. You do not need to give the values of the function at the points.

Problem 6 (20 points)

Let $f(x, y, z) = x^2 y + (x - y)\cos(\pi z)$.

(a) Find the gradient of f at the point $(1, 2, 1)$.

(b) In what direction is the directional derivative of f at $(1, 2, 1)$ the largest?
What is the derivative in that direction, at that point?

(c) What is the derivative of f at $(1, 2, 1)$ in the direction of the vector $v = 2\vec{\imath} + \vec{\jmath} - 2\vec{k}$?

Problem 7 (20 points)

Use Stokes' theorem to evaluate $\oint_C \overrightarrow{F} \cdot \overrightarrow{T}\, ds$ where

(i) $\overrightarrow{F}(x, y, z) = y\vec{\imath} - x\vec{\jmath} + 3\vec{k}$.

(ii) C is the circle where the plane $2x + 2y + 2z = 0$ intersects the sphere $x^2 + y^2 + z^2 = 9$.

Problem 8 (20 points)

Use Green's Theorem to evaluate the integral $\int_C x^2 y\,dx - 3y^2\,dy$

where C is the circle $x^2 + y^2 = 1$, oriented counter-clockwise.

Problem 9 (20 points)

For motion along the curve given parametrically by $\vec{r}(t) = t^2\vec{\imath} + \frac{2}{3}t^3\vec{\jmath} + t\vec{k}$:

(a) Find the velocity $\vec{v}(t)$ at the point where $t = 1$

(b) Find the acceleration $\vec{a}(t)$ at the point where $t = 1$

(c) Find the unit tangent vector $\overrightarrow{T}(t)$ at the point where $t = 1$

(d) Find the principal unit normal vector $\overrightarrow{N}(t)$ at the point where $t = 1$

(e) Find the curvature $\kappa(t)$ at the point where $t = 1$

Problem 10 (20 points)

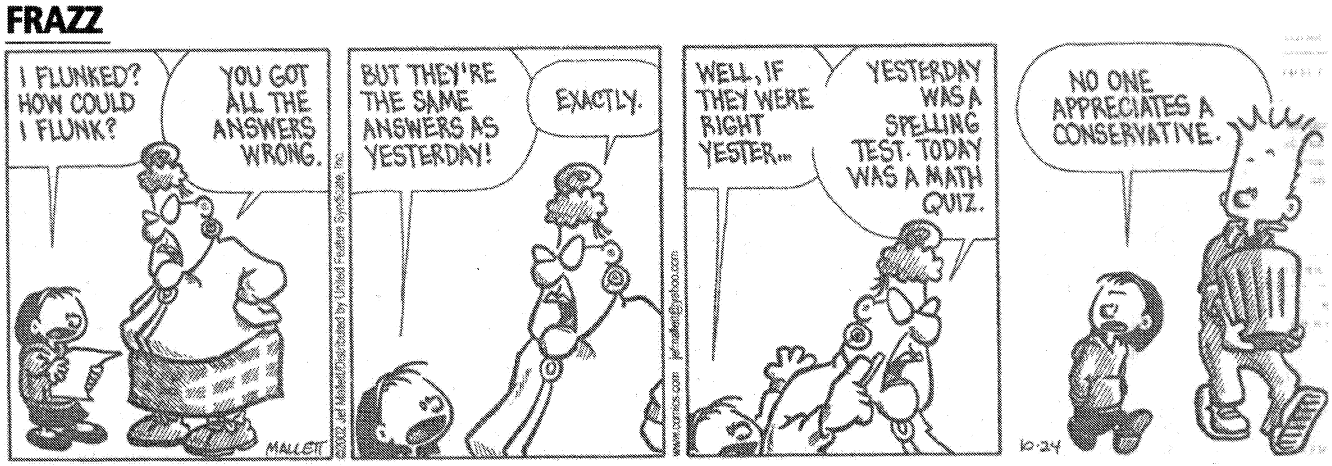
$$\text{Let } f(x,y) = \frac{xy}{x^2+y^2}.$$

Show that

$$\lim_{(x,y)\to(0,0)} f(x,y)$$

does not exist.

Hint: There are lines other than the coordinate axes.



Scratch Paper