Your Name: _____

Circle your TA's name:

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Mathematics 234, Fall 2005 Lecture 2 (Wilson)

First Midterm Exam October 6, 2005

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is a problem on the back of this sheet: Be sure not to skip over it by accident!

There is scratch paper at the end of this exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought on an index card, as announced in class and on the class website.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)

Problem	Points	Score
1	12	
2	14	
3	12	
4	10	
5	12	
6	12	
7	14	
8	14	
TOTAL	100	

Problem 1 (12 points)

Let $f(x, y) = x\cos(y) + e^{xy}$ and let P be the point $(2, 0)$.

(a) Find the gradient $\overrightarrow{\nabla f}$ at P.

(b) For the vector $\vec{v} = -3\vec{\imath} + 4\vec{\jmath}$, find the directional derivative of f at P in the direction of $\vec{v}$.

(c) Find a vector $\vec{w}$ tangent to the level curve $f(x, y) = 3$ at P.

(d) Give a vector in the direction which makes the directional derivative at P largest. What is the directional derivative in that direction?

Problem 2 (14 points)

(a) The function $f(x,y) = \frac{x^2-2xy+4y}{x^2-y^2}$ is continuous at the point $(1,2)$.
How can I tell that is true? Give reasons based on theorems in the textbook: You do not need to quote the theorems or cite where a theorem appears in the book.

What is $\lim_{(x,y)\to(1,2)} f(x,y)$?

(b) The function $f(x,y) = \frac{2xy}{x^2+y^2}$ does not have a limit as $(x,y) \to (0,0)$.
Justify this statement carefully: Give numeric evidence, don't just paraphrase the definition of the limit.

Problem 3 (12 points)

Some mechanism forces an object to move with position vector given by $\vec{r}(t) = t^2\vec{\imath} + e^t\vec{\jmath} + t\,e^t\vec{k}$, for $-1 \leq t \leq 1$.

(a) Find the velocity for this motion, both in general and at $t = 0$.

(b) Find the acceleration for this motion, both in general and at $t = 0$.

(c) Find equations for the tangent line to this object's path at $t = 0$.

(d) Suppose the mechanism breaks and from the instant $t = 0$ the object flies freely. Will it ever hit the plane where $y = z$? If so, at what point will it hit that plane?

Problem 4 (10 points)

Let $\vec{r}(t) = 12\sin(t)\vec{\imath} + 5t\,\vec{\jmath} + 12\cos(t)\vec{k}$ describe the motion of an object along a curve in space.

Find as functions of t:

(a) The velocity $\vec{v}(t)$

(b) The acceleration $\vec{a}(t)$

(c) The unit tangent vector $\vec{T}(t)$

(d) The principal unit normal vector $\vec{N}(t)$

(e) The curvature $\kappa(t)$

(f) The tangential (scalar) component of acceleration a_T

(g) The normal (scalar) component of acceleration a_N

You may find them in whatever order is convenient for you, but Be sure to label each answer!

Problem 5 (12 points)
For the function $f(x, y) = x^2 - xy + 3y^2$:

(a) Find an equation for the tangent plane to the graph of $f(x, y)$ at the point $(3, -1, 15)$.

(b) Use a linear approximation to estimate $f(2.98, -0.97)$.

Do not just calculate $f(2.98, -0.97)$ by plugging those two numbers into the formula for f!

Problem 6 (12 points)
Let $f(x, y, z) = xyz + x\sin(z)$.

(a) Calculate all three first partial derivatives of f.
Be sure to label which is which!

(b) Calculate all nine second partial derivatives of f.
If you use a theorem to reduce the number of distinct calculations, say where it is used.
Be sure to label which derivative is which!

Problem 7 (14 points)

Let $f(x, y) = x^2 + 2x + y^2 - 3y + 4$.

Find all points (x, y) such that the tangent plane to the surface $z = f(x, y)$ at the point $(x, y, f(x, y))$ is parallel to the plane $-2x + 3y - z = 0$.

Hint: Write an equation involving x_0 and y_0 that describes the tangent plane to the surface at the particular point $(x_0, y_o, f(x_0, y_0))$. Remember how the equations of parallel planes compare, and solve for which value(s) of x_0 and y_0 make these parallel.

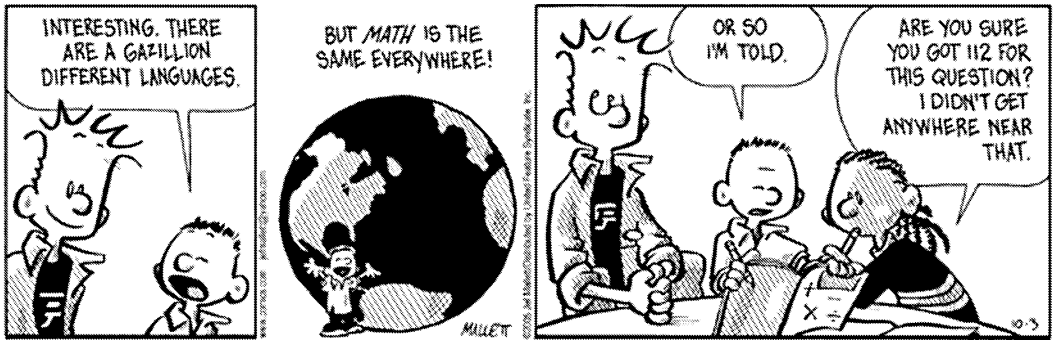
Problem 8 (14 points)

(a) Consider the function given in cylindrical coordinates $F(r, \theta, z) = 2r^2 + z^2$. Convert the function to rectangular (x-y-z) coordinates and find an equation for the tangent plane to the level surface through the point (in rectangular coordinates) (1,2,3).

(b) An object moves in air so that its position (x, y, z) at time t is given by $(\sin(t), 2\cos(t), e^{3t})$.

The region of space through which the object moves has air temperature given by $T(x, y, x) = 20 + xyz - x^2 - 2y$ degrees on some scale.

Find $\frac{dT}{dt}$, both for t in general and at the instant when $t = \pi$.



Scratch Paper

(not a command!)