Your Name: ______________________________

Circle your TA's name:

Jesse Holzer **Evan (Alec) Johnson** **Asher Kach** **Liming Lin**

Mathematics 234, Fall 2004 **Lecture 3 (Wilson)**

Second Midterm Exam November 15, 2004

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is a problem on the back of this sheet: Be sure not to skip over it by accident!

You may refer to notes you have brought on an index card, as announced in class and on the class website.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)

Problem	Points	Score
1	12	
2	12	
3	12	
4	12	
5	14	
6	12	
7	12	
8	14	
TOTAL	100	

Problem 1 (12 points)

(a) Set up but do not evaluate an integral in spherical coordinates to compute the volume of the part of the sphere of radius 3, centered at the origin, where z is non-negative. (I.e. the region in the sphere $x^2 + y^2 + z^2 \leq 9$ which is above the xy-plane.)

(b) Evaluate the triple integral $\int_0^{\pi} \int_0^{2\theta} \int_0^{\sqrt{4-r^2}} 2\, r\, z \; dz\; dr\; d\theta$

Problem 2 (12 points)
The volume of a cylindrical can of radius r and height h is $V(r,h) = \pi r^2 h$.
A certain can is intended to have a radius of $r = 2$ inches and a height of $h = 5$ inches. Due to manufacturing tolerances, the radius changes to $r = 2.01$ inches and the height changes to $h = 4.98$ inches.
Use partial derivatives to approximate how much the volume of the can changes from its designed value.
(Do not just compute the two volumes, with or without a calculator, and subtract!)

Problem 3 (12 points)

Find an equation for the tangent plane to the surface $x^3 + 3xyz + 2y^3 - z^3 = -15$ at the point $(1, -1, 2)$.

Problem 4 (12 points)

(a) Compute $\iiint\limits_R (xz + 2z)\, dV$, where R is the region in the first octant bounded by $x^2 + z^2 = 4$ and $x + y = 2$.

(b) Convert the integral $\int_{-2}^{0} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (x^2 + y^2 + 2x)\, dy\, dx$ to polar coordinates. You do not have to evaluate this integral.

Problem 5 (14 points)

Find and identify all relative maxima, relative minima, and saddle points, for the function $f(x, y) = x^3 + y^2 - 6x^2 + y - 1$.

Problem 6 (12 points)
Set up but do not evaluate an integral to compute the surface area for the section of the sphere $x^2 + y^2 + z^2 = 4$ inside the vertical cylinder $x^2 + (y-1)^2 = 1$.

Problem 7 (12 points)
A thin plate covers the triangular region in the plane whose edges are the x-axis, the line $y = 2x$, and the line $x = 1$.
The density of this plate is given by the function $\delta(x, y) = 6x + 6y + 6$.

(a) Use an integral to evaluate the mass of this plate.

(b) Find the moment M_x of the plate about the x-axis.

(c) Find the moment M_y of the plate about the y-axis.

(d) Find the coordinates $(\overline{x}, \overline{y})$ of the center of mass of this plate.

(e) Find the moment of inertia (second moment) of this plate about the x-axis, I_x.

(f) Find the radius of gyration of this plate about the x-axis.

Problem 8 (14 points)
Find the largest and smallest values of $f(x,y) = x^2 + y$ subject to $x^2 + y^2 = 4$.