Your Name: ______________________________

Circle your TA's name:

Jesse Holzer **Evan (Alec) Johnson** **Asher Kach** **Liming Lin**

Mathematics 234, Fall 2004 **Lecture 3 (Wilson)**

First Midterm Exam October 4, 2004

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is a problem on the back of this sheet: Be sure not to skip over it by accident!

There is scratch paper at the end of this exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought on an index card, as announced in class and on the class website.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)

Problem	Points	Score
1	10	
2	20	
3	12	
4	12	
5	12	
6	12	
7	10	
8	12	

Problem 1 (10 points)

In class we saw pictures of the helix (corkscrew) given by the position vector

$$\vec{r}(t) = \sin(t)\vec{\imath} + t\,\vec{\jmath} + \cos(t)\vec{k}.$$

Calculate the arclength of one turn of this helix, from $t = 0$ to $t = 2\pi$.

<u>Problem 2</u> (20 points)

Let $\vec{r}(t) = 3\sin(t)\vec{\imath} + 3\cos(t)\vec{\jmath} + 4t\,\vec{k}$ describe the motion of an object along a curve in space. Find as functions of t:

(a) The velocity $\vec{v}(t)$

(b) The acceleration $\vec{a}(t)$

(c) The unit tangent vector $\vec{T}(t)$

(d) The principal unit normal vector $\vec{N}(t)$

(e) The curvature $\kappa(t)$

(f) The tangential (scalar) component of acceleration a_T

(g) The normal (scalar) component of acceleration a_N

Be sure to label each answer! (You may have additional space at the bottom of the opposite page, Problem 1, to use for this problem.)

Problem 3 (12 points)

For each of the four descriptions, fill in A or B or C or D to indicate which graph below it corresponds to.

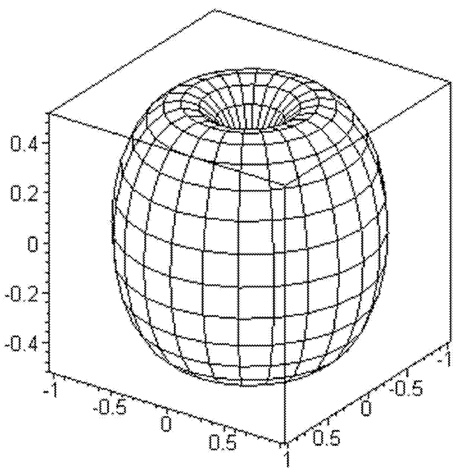
(i) $z = y\sin(x)$ ____________ (answer)

(ii) $r = 1 + \cos(\theta)$, any value of z ____________ (answer)

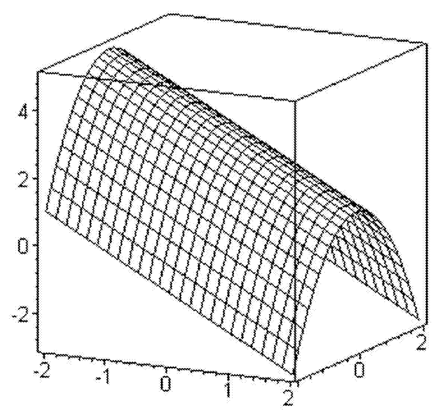
(iii) $z = 3 - x^2 - y$ ____________ (answer)

(iv) $\rho = \sin(\phi)$, any value of θ ____________ (answer)

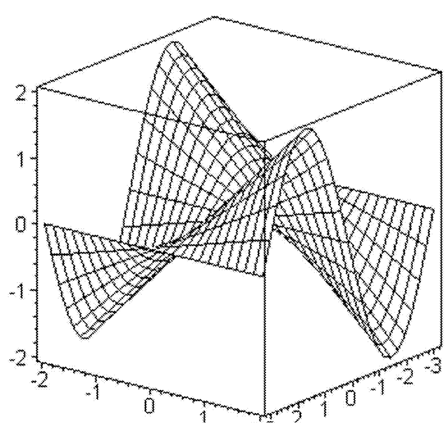
A



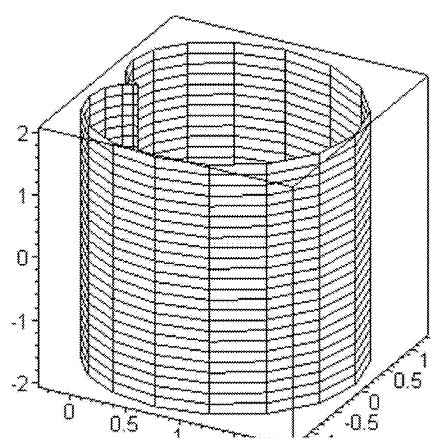
B



C



D



<u>Problem 4</u> (12 points)

For $f(x,y) = \sin(x^2 + 2y)$, find:

(a) $\dfrac{\partial f}{\partial x}$

(b) $\dfrac{\partial f}{\partial y}$

(c) $\dfrac{\partial^2 f}{\partial x^2}$

(d) $\dfrac{\partial^2 f}{\partial y \partial x}$

(e) $\dfrac{\partial^2 f}{\partial x \partial y}$

(f) $\dfrac{\partial^2 f}{\partial y^2}$

Problem 5 (12 points)

(a) Evaluate $\lim_{(x,y)\to(2,\pi)} x^2\cos(2y)$.

(b) Show that $\lim_{(x,y)\to(0,0)} \dfrac{x}{\sqrt{x^2+y^2}}$ does not exist.

Problem 6 (12 points)

Let $f(x, y) = x^2 e^y$.

Let P be the point $(1, 0)$.

(a) Find the gradient ∇f at the point P.

(b) Find the directional derivative of f at P in the direction of an arbitrary vector $\vec{v} = v_1\vec{\imath} + v_2\vec{\jmath}$.

(c) Find the directional derivative of f at P in the direction from $(1, 0)$ to $(4, 4)$.

(d) In what direction is the directional derivative of f at P largest? What is the directional derivative in that direction?

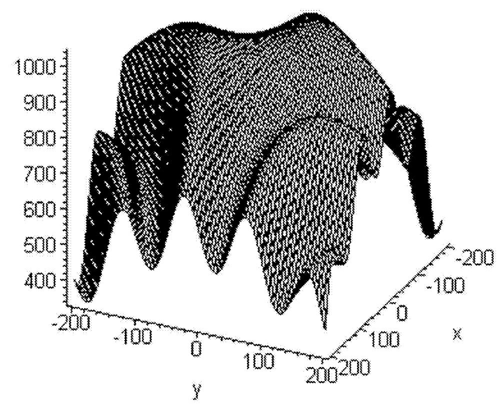
Problem 7 (10 points)

Suppose $w = \sin(xy) + x\sin(y)$, where $x = u^2 + v^2$ and $y = 2u + v - 2$. Using the chain rule:

(a) Find $\dfrac{\partial w}{\partial u}$.

(b) Find $\dfrac{\partial w}{\partial v}$.

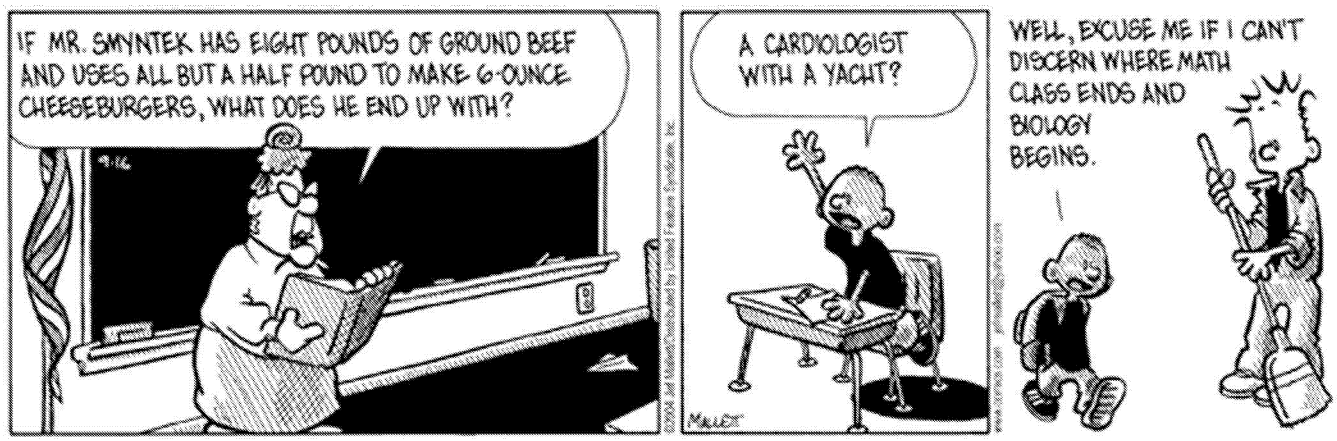
Problem 8 (12 points)



Suppose you are walking over some hills. The north-south direction is measured as y and the east-west direction as x. The altitude in feet is given by

$$f(x,y) = 1000 - 100\sin\left(\frac{xy}{3000}\right) - \frac{x^2}{100} - \frac{y^2}{200}.$$

If you are at the point where $x = 100$ and $y = -100$, and you start walking toward the center where $x = 0$ and $y = 0$, will you begin by walking uphill, downhill, or horizontally? Be sure to show how you determine your answer.



Scratch Paper

(not a command!)