Mathematics 223, Lecture 3 (Wilson)   Your Name: ____________________

Circle your TA's name:

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Final Exam   5/17/94

Write your answers to the twelve problems in the spaces provided.
When a problem says to use a particular technique to solve a differential equation, you may receive NO credit unless your answer shows that you can implement that technique.
There is scratch paper at the end of the exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on upt to four 4" by 6" index card, as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	14	
2	16	
3	20	
4	16	
5	15	
6	16	
7	18	
8	16	
9	19	
10	18	
11	16	
12	16	
TOTAL	200	

Problem 1 (14 points)
Let $f(x,y,z) = xy + 3yz^2$. Find both the tangent plane and the normal line to the level surface $f(x,y,z) = 3$, at the point $P_0 = (0,1,-1)$.

Represent the tangent plane by an equation of the form $Ax + By + Cz = D$, and the normal line in parametric form $x = a_1t + b_1$, $y = a_2t + b_2$, and $z = a_3t + b_3$.

Problem 2 (16 points)
(You may leave your answers to this problem in implicit form.)

(a) Find all solutions of

$$\frac{dy}{dx} = \frac{e^{y^2+\sin x} \ \cos x}{y}$$

(b) Find the solution to the equation above which satisfies $y(\frac{\pi}{2}) = -1$.

Problem 3 (20 points)

Find the largest and smallest values of the function $f(x, y, z) = 6x + 3y + 2z - 5$ on the ellipsoid $4x^2 + 2y^2 + z^2 = 70$, and the points on the ellipsoid where those values occur.

Problem 4 (16 points)

(a) Find a vector field $\vec{F} = M(x,y)\vec{i} + N(x,y)\vec{j}$ which is zero at the origin and at any other point (a,b) in the plane points directly away from the origin with magnitude $|\vec{F}| = 4\sqrt{a^2+b^2}$.

(b) Compute the flux of the field from (a) across the ellipse $\vec{r} = 3\cos(t)\vec{i} + 4\sin(t)\vec{j}$ for $0 \le t \le 2\pi$.

Problem 5 (15 points)
Use Taylor's Formula to construct a polynomial of degree 2 which approximates

$$f(x, y) = \frac{1}{2x - 3y + 1}$$

near the origin.

Problem 6 (16 points)

Evaluate the line integral of $f(x, y, z) = \sqrt{x^2 + y^2}$ along the curve $\vec{r} = (4\cos t)\vec{i} + (4\sin t)\vec{j} + 3t\vec{k}$ for $-2\pi \leq t \leq 2\pi$.

Problem 7 (18 points)
Let $f(x,y,z) = x^2e^y + y^3z$.

(a) Find the direction in which f is most rapidly increasing at $P_0 = (2,0,1)$, and the derivative of f in that direction.

(b) Use a linear approximation to f to find approximately the value of $f(1.9, 0.2, 1.1)$.

Problem 8 (16 points)
Let $f(x, y) = 2x^3 - 15x^2 + 36x + 2y^3 + 3y^2 - 12y$.
Find all critical points of $f(x)$. For each critical point tell whether it is a local or absolute maximum, a local or absolute minimum, or a saddle point for $f(x)$.

Problem 9 (19 points)

Write and evaluate a triple integral, using cylindrical coordinates, for the integral of $f(x, y, z) = 6 + 4x$ through the region in the first octant (*i.e.* where x, y, and z are all at least zero) which is bounded by the cone $x = \sqrt{y^2 + z^2}$, the cylinder $y^2 + z^2 = 1$, and the coordinate planes.

Problem 10 (18 points)

(a) Find all solutions to

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x^2$$

(b) Find the solution to the equation in (a) which satisfies $y(0) = 0$ and $y(1) = 3$.

Problem 11 (16 points)
Find all solutions to

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$$

Write the solutions as combinations of *real* functions, *i.e.* if your solution has explicit imaginary numbers in it, rewrite it in another form.

Problem 12 (16 points)

(a) Find all solutions of

$$x\frac{dy}{dx} + 2y = x^3$$

(b) Find the solution of the equation in (a) which satisfies $y(2) = 1$.

SCRATCH PAPER