Mathematics 223, Lecture 3 (Wilson) Your Name: ____________________

Circle your TA's name:

Nicolas Eisen Kellie Evans Jonathan Kochavi

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Exam II 4/7/94

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using π, e, $\sqrt{3}$, $\ln(2)$, and similar numbers) rather than using decimal approximations.

There is scratch paper at the end of the exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on one 4" by 6" index card, as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	13	
2	12	
3	14	
4	16	
5	12	
6	12	
7	11	
8	10	
TOTAL	100	

Problem 1 (13 points)
Find the area, the mass, and the center of mass of a thin triangular plate which covers the region bounded by the lines $y = 3x$, $x = 2$, and $y = 0$, and with density at the point (x, y) given by $\delta(x, y) = xy + y + 1$.

(Use at least one integral in *each* of the three calculations.)

Problem 2 (12 points)
Use an integral to compute the volume of the solid which
(a) is inside the vertical cylinder $x^2 + y^2 = 4$,
(b) is above the $x - y$ plane, and
(c) is under the horizontal cylinder $z = 4 - x^2$.

(You may wish to consider what coordinate system will be most convenient...)

Problem 3 (14 points)
Evaluate the integral

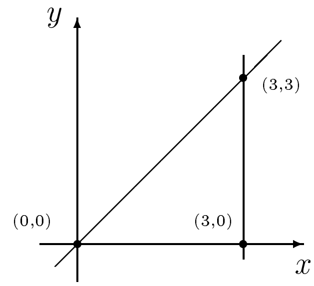
$$\int_0^2 \int_{\frac{y}{2}}^{\frac{y+4}{2}} y^3 \ (2x - y) \ e^{(2x-y)^2} dx \ dy$$

using the transformation $x = u + \frac{v}{2}$, $y = v$.

Problem 4 (16 points)
(a) Find a vector field $\vec{F} = M(x,y)\vec{\imath} + N(x,y)\vec{\jmath}$ such that:
At the origin, $\vec{F}$ is zero.
At any point (a,b) in the plane, $\vec{F}$ points along the tangent to the circle $x^2 + y^2 = a^2 + b^2$ in the *clockwise* direction, with magnitude $|\vec{F}| = 4\sqrt{a^2+b^2}$.

(b) Find the circulation of this field around the ellipse $\vec{r} = 4\cos(t)\vec{\imath} + 2\sin(t)\vec{\jmath},\ \ 0 \le t \le 2\pi$.

Problem 5 (12 points)
Let C be the closed curve which is the boundary of the triangle shown in the picture.



Use Green's Theorem to find both the counterclockwise circulation around C and the outward flux across C for the field $\vec{F} = (y^2 - x^2)\vec{i} + (x^2 + y^2)\vec{j}$.

Problem 6 (12 points)

Find the work done by the force field $\vec{F} = \left(\frac{1}{x^2+1}\right)\vec{j}$ in moving along the curve $\vec{r} = t\vec{i} + t^2\vec{j} + t^4\vec{k}$ for $0 \leq t \leq 1$.

Problem 7 (11 points)

Evaluate the line integral of $f(x, y, z) = \sqrt{x^2 + y^2}$ along the curve $\vec{r} = 3\cos(t)\vec{i} + 3\sin(t)\vec{j} + 4t\vec{k}$, for $0 \leq t \leq 2\pi$.

Problem 8 (10 points)

Set up *but do not evaluate* an integral equivalent to the following but with the order of integration reversed:

$$\int_0^2 \int_0^{8-2y^2} (3x^2 - y)\, dx\, dy$$

SCRATCH PAPER