Mathematics 223, Lecture 4 (Wilson) Your Name: ____________

Circle your TA's name:

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Final Exam 12/19/95

Write your answers to the eleven problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\pi$, $e$, $\sqrt{3}$, $\ln(2)$, and similar numbers) rather than using decimal approximations.

You may refer to notes you have brought in on up to three sheets of paper, as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. THIS APPLIES ESPECIALLY TO PROBLEMS ON WHICH YOU MAKE EXTENSIVE USE OF YOUR CALCULATOR: TELL EXACTLY WHAT YOU ASKED IT TO DO, NOT JUST THE RESULT.

| Problem | Points | Score |
|---|---|---|
| 1 | 20 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 20 | |
| 5 | 18 | |
| 6 | 18 | |
| 7 | 18 | |
| 8 | 14 | |
| 9 | 16 | |
| 10 | 18 | |
| 11 | 18 | |
| TOTAL | 200 | |

Problem 1 (20 points)

Find the volume of the solid region which is:
Under the surface $z = x^2 + 2y^2 + 1$ and above $z = 0$,
Has one side determined by the plane $x = 0$,
Has one side determined by the plane $y = 0$, and
Has one side determined by the plane $3x + 2y = 6$.

Problem 2 (20 points)

Let $\vec{F}$ be the vector field given by $\vec{F}(x, y, z) = 2y\vec{i} + 3x\vec{j} + (x + y)\vec{k}$. Find the work done by $\vec{F}$ in moving along the path $\vec{r} = (\cos t)\vec{i} + (\sin t)\vec{j} + (t/6)\vec{k}$ for $t$ going from 0 to $\pi$.

Problem 3 (20 points)

Let $f(x, y, z) = 2xy + x^3 - z^2$. Let $P_0 = (1, -1, 3)$.

(a) Find $\overrightarrow{\nabla f}$.

(b) Find $\overrightarrow{\nabla f}\ |_{P_0}$.

(c) Find a <u>unit</u> vector in the direction in which $f$ is increasing most rapidly at $P_0$.

(d) Find the derivative of $f$ in the direction of the vector from (c).

(e) Find a non-zero vector $\vec{v}$ such that the directional derivative of $f$ at $P_0$ in the direction of $\vec{v}$ is zero.

Problem 4 (20 points)

(a) Find all solutions of $y'' - 4y' + 13y = 0$.

(b) Find the solution of $y'' - 6y' + 9y = 0$ which satisfies $y(0) = 2$ and $y'(0) = -7$.

Problem 5 (18 points)

Find the solution of

$$\frac{dy}{dx} = y^2(2x - \sin x)$$

which satisfies $y(0) = 2$.

Problem 6 (18 points)

Let

$$f(x, y) = 2x^3 - 3x^2 - 12x + y^3 - 6y^2 + 9y - 1$$

Find all local maxima, local minima, and saddle points of $f$.

Problem 7 (18 points)

Use Green's theorem to evaluate the outward flux of the field $\vec{F} = 4xy\ \vec{\imath} + (2y + x)\ \vec{\jmath}$ across the curve $C$ which is the boundary of the triangle with sides $x = 0$, $y = 0$, and $y = 2 - 2x$.

Problem 8 (14 points)

A first order differential equation has the direction field shown, where the vertical axis gives the dependent variable $y(x)$ and the slope of each line segment is the value of $y'(x)$ at the left end of the segment. You may assume the slopes to the left and right of the region pictured follow the trends shown here.

(a) If a solution of this differential equation satisfies the initial condition $y(0) = 1$, approximately what value must $y(1)$ have? WHY?

(b) If $\phi(x)$ is any solution of this equation (not necessarily satisfying the initial condition in (a)!) what can you say about the behavior of $\phi(x)$ as $x \to \infty$?

Problem 9 (16 points)

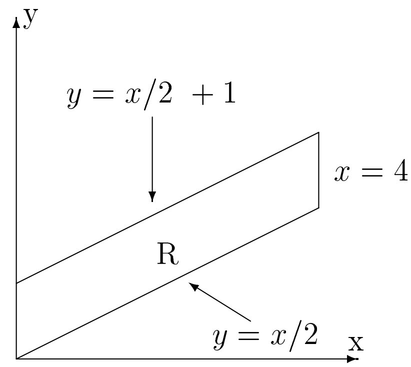
Let $f(x,y) = x^2y + e^x$, $x(u,v) = 2u^2 - 3v^2$, and $y(u,v) = 3uv$.

(a) Find $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$. You may leave the variables $x, y, u, v$ in your answer.

(b) Find $\frac{\partial f}{\partial u}$ at the point $(u,v) = (1,0)$. This should turn out to be a number, with no $x, y, u, v$ remaining.

Problem 10 (18 points)

Let $R$ be the parallelogram-shaped region in the picture:




Use the transformation

$$u = y - \frac{x}{2} \qquad v = \frac{x}{2}$$

to evaluate the double integral of the function

$$f(x) = \frac{2y - x}{2}$$

over $R$.

Problem 11 (18 points)

(a) Find all solutions of $y'' - 6y' + 9y = 0$.

(b) Find all solutions of $y'' - 6y' + 9y = e^x$.