Mathematics 222 **Summer 2002** **Wilson**

Your Name: ______________________________

Please circle the name of your discussion instructor:

Adrian Jenkins **Bob Wilson**

Final Exam August 8, 2002
Morning Portion

- **Write your answers to the six problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.**
- **On the other side of this sheet there is a collection of facts and formulas.**
- **Wherever applicable, leave your answers in exact forms (using π, e, $\sqrt{3}$, $\ln(2)$, and similar numbers) rather than using decimal approximations.**
- **You may refer to notes you have brought in on one or two sheets of paper as announced in class.**

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" are not sufficient substantiation...)

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
TOTAL	120	

Some formulas, identities, limits, and numeric values you might find useful:

Values of trig functions:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	—

Trig facts:

1. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
2. $\sec\theta = \frac{1}{\cos\theta}$
3. $\sin^2\theta + \cos^2\theta = 1$
4. $\sec^2\theta = \tan^2\theta + 1$
5. $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
6. $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
7. $\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$
8. $\sin^2 x = \frac{1}{2}(1-\cos 2x)$
9. $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

Derivative formulas:

1. $\frac{d}{dx}\tan x = \sec^2 x$
2. $\frac{d}{dx}\sec x = \sec x\,\tan x$
3. $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
4. $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$
5. $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$
6. $\frac{d}{dx}\ln x = \frac{1}{x}$
7. $\frac{d}{dx}e^x = e^x$

Integral formulas: (We assume you know, and you are certainly allowed to use, basic formulas for integrals of functions such as x^n, e^x, $\sin x$, $\cos x$, etc., and how to use substitution to extend these.)

1. $\int \frac{1}{u}\,du = \ln|u| + C$
2. $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$
3. $\int \frac{du}{1+u^2} = \tan^{-1} u + C$
4. $\int \sec(u)\,du = \ln|\sec(u) + \tan(u)| + C$
5. $\int u\,dv = uv - \int v\,du$

Algebra formulas:

1. $a^{x+y} = a^x\,a^y$
2. $a^b = e^{b\ln a}$
3. $\ln(xy) = \ln(x) + \ln(y)$

Some commonly encountered limits:

1. $\lim_{n\to\infty} \frac{\ln n}{n} = 0$
2. $\lim_{n\to\infty} \sqrt[n]{n} = 1$
3. $\lim_{n\to\infty} x^{\frac{1}{n}} = 1 \quad (x>0)$
4. $\lim_{n\to\infty} x^n = 0 \quad (|x|<1)$
5. $\lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n = e^x \quad (\text{any } x)$
6. $\lim_{n\to\infty} \frac{x^n}{n!} = 0 \quad (\text{any } x)$

Problem 1 (20 points)

(a) Find the solution to $y' + 3x^2y = e^{2x-x^3}$ that satisfies $y(0) = \frac{5}{2}$.

(b) What will be the value of $y(1)$? (The answer should be, in some form, a number. You do not have to give it as a decimal fraction, though.)

Problem 2 (20 points)

For each of the following series, tell all you can about its convergence. This should include whether it converges or diverges, and if appropriate whether it converges absolutely or conditionally. Be sure to give reasons for your answers!

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\left(\frac{\ln(n)}{n}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty}\cos\left(\frac{4}{n}\right)$

(c) $\displaystyle\sum_{n=1}^{\infty}\frac{3}{5^n}$

(d) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\left(\frac{2}{n\sqrt{n}}\right)$

Problem 3 (20 points)

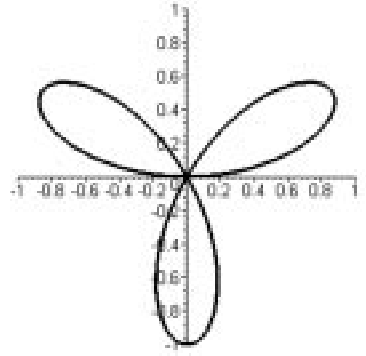
Consider the curve given parametrically as $x = -2 - \sin 2t$ and $y = 3 + 3\cos t$ for $0 \le t \le 2\pi$.

(a) Describe the path this traverses: Where does it start (when $t = 0$), how does it progress and along what path, and where does it end? (The path is one of the conic sections we have studied. You should identify it as such.)

(b) Give an equation in rectangular (Cartesian) form relating x and y without the parameter t. (You may find the identity $\sin^2 t + \cos^2 t = 1$ useful in deriving this equation.)

Problem 4 (20 points)

Find the area inside <u>one leaf</u> of the three-leafed rose $r = \sin(3\theta)$. Be sure to show all work: Do not assume you can read numbers off of the picture at the right, which is included just to help you see what the region looks like. In particular, show calculations used to determine any significant angles!



Problem 5 (20 points)
Use words to describe the following sets of points in space.

(a) $y = 0$ and $z \geq |x|$

(b) $4 \leq x^2 + y^2 + z^2 \leq 9$

(c) $\frac{\pi}{2} \leq \phi \leq \pi$ and $0 \leq \theta \leq \frac{\pi}{2}$.

(d) $r = z^2$

Problem 6 (20 points)
Let $\vec{v} = \vec{i} + 2\vec{j} - \vec{k}$ and $\vec{u} = 8\vec{i} + 4\vec{j} - 12\vec{k}$.

(a) Find the (scalar) component of $\vec{u}$ in the direction of $\vec{v}$.

(b) Find the (vector) projection of $\vec{u}$ on $\vec{v}$.

(c) Write $\vec{u}$ as a sum of two vectors, one parallel to $\vec{v}$ and one orthogonal to $\vec{v}$.