Mathematics 222 **Summer 2002** **Wilson**

Your Name: ______________________________

Please circle the name of your discussion instructor:

Adrian Jenkins **Bob Wilson**

Final Exam August 8, 2002
Afternoon Portion

- **Write your answers to the four problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.**
- **On the other side of this sheet there is a collection of facts and formulas.**
- **Wherever applicable, leave your answers in exact forms (using π, e, $\sqrt{3}$, $\ln(2)$, and similar numbers) rather than using decimal approximations.**
- **You may refer to notes you have brought in on one or two sheets of paper as announced in class.**

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" are not sufficient substantiation...)

Problem	Points	Score
7	20	
8	20	
9	20	
10	20	
TOTAL	80	

Some formulas, identities, limits, and numeric values you might find useful:

Values of trig functions:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	—

Trig facts:

1. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
2. $\sec\theta = \frac{1}{\cos\theta}$
3. $\sin^2\theta + \cos^2\theta = 1$
4. $\sec^2\theta = \tan^2\theta + 1$
5. $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
6. $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
7. $\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$
8. $\sin^2 x = \frac{1}{2}(1-\cos 2x)$
9. $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

Derivative formulas:

1. $\frac{d}{dx}\tan x = \sec^2 x$
2. $\frac{d}{dx}\sec x = \sec x\,\tan x$
3. $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
4. $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$
5. $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$
6. $\frac{d}{dx}\ln x = \frac{1}{x}$
7. $\frac{d}{dx}e^x = e^x$

Integral formulas: (We assume you know, and you are certainly allowed to use, basic formulas for integrals of functions such as x^n, e^x, $\sin x$, $\cos x$, etc., and how to use substitution to extend these.)

1. $\int \frac{1}{u}\,du = \ln|u| + C$
2. $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$
3. $\int \frac{du}{1+u^2} = \tan^{-1} u + C$
4. $\int \sec(u)\,du = \ln|\sec(u) + \tan(u)| + C$
5. $\int u\,dv = uv - \int v\,du$

Algebra formulas:

1. $a^{x+y} = a^x a^y$
2. $a^b = e^{b\ln a}$
3. $\ln(xy) = \ln(x) + \ln(y)$

Some commonly encountered limits:

1. $\lim_{n\to\infty} \frac{\ln n}{n} = 0$
2. $\lim_{n\to\infty} \sqrt[n]{n} = 1$
3. $\lim_{n\to\infty} x^{\frac{1}{n}} = 1 \quad (x > 0)$
4. $\lim_{n\to\infty} x^n = 0 \quad (|x| < 1)$
5. $\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x \quad (\text{any } x)$
6. $\lim_{n\to\infty} \frac{x^n}{n!} = 0 \quad (\text{any } x)$

Problem 7 (20 points)
Let $A = (1, 2, -1)$, $B = (2, 2, 0)$, and $C = (1, 3, 1)$ be three points in space in ordinary rectangular (Cartesian) coordinates.

(a) What is the area of the triangle whose vertices are A, B, and C?

(b) Find a vector of unit length perpendicular to the plane containing these three points.

(c) Find an equation for the plane containing these three points.

(d) Find equations for the line through B perpendicular to the plane containing the three points.

Problem 8 (20 points)
Consider the graph of

$$9x^2 - 16y^2 - 36x - 128y - 364 = 0.$$

(a) For some translated coordinates x' and y', the center (or vertex if this is a parabola) will be at the origin of the coordinate system. Write the equation in x' and y'.

(b) Is this graph a circle, an ellipse, a parabola, or a hyperbola?

(c) If it is a circle, ellipse, or hyperbola, where is its center? If it is a parabola, where is its vertex? (Be sure to use (x, y) coordinates for this and the following answers!)

(d) (i) If it is a circle, what is the radius? What are the coordinates of two points on the circle?

(ii) If it is an ellipse, where are the ends of the major axis (the two points furthest from the center)? Where are the foci? What is the eccentricity?

(iii) If it is a parabola, where is the focus? Give an equation for the directrix.

(iv) If it is a hyperbola, where are the foci? Where does it cross the x' or y' axes (the translated axes you found in (a)). (You can identify these points using either coordinates but make clear which you are using.) Give equations (in x and y) for the asymptotes.

Problem 9 (20 points)
Some initial terms of the Maclaurin series for e^{-x} are used to approximate $f(x) = e^{-x}$, for $-0.1 < x < 0.1$. How many terms are needed in order to guarantee that the approximation is within ± 0.0001 of the actual value of $f(x)$ for all x in that range? Write out the polynomial showing those terms.
Your answer should both tell the number of terms needed (or the degree of the highest power term needed, but be sure to indicate which!) and the polynomial you would use as your approximation.

(You may use the fact that $|e^x|$ and $|e^{-x}|$ are both less than 2 for $-0.1 < x < 0.1$.)

Problem 10 (20 points)

Find all solutions of $y'' + 2y' = 4x + 3$.

(a) Find all solutions of $y'' + 2y' = 0$:

(b) Find a solution of $y'' + 2y' = 4x + 3$:

(c) Write an expression for all solutions of $y'' + 2y' = 4x + 3$: