Mathematics 222, Summer 2001 Lecture 1 (Wilson)

Final Exam August 9, 2001

Your Name: ____________________

Circle your discussion instructor's name:

Ramiro de la Vega Bob Wilson

THERE IS ARE PROBLEMS ON THE BACK OF THIS SHEET!

There are two parts to this exam. The first six problems are for you to work on this morning: You will receive the remaining four problems this afternoon. Write your answers in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using π, $\sqrt{3}$, $\cos^{-1}(1/5)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

If you need scratch paper, or additional paper to write your answers on, please ask for it.

You may refer to notes you have brought in, as announced in class.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" is not sufficient substantiation...)

Problem	Points	Score
1	21	
2	20	
3	20	
4	18	
5	18	
6	21	
7	22	
8	20	
9	20	
10	20	
TOTAL	200	

Problem 1 (21 points)
Solve the initial value problem:

$$2y'' - y' - y = 0 \qquad \text{and} \qquad y(0) = -1 \qquad \text{and} \qquad y'(0) = 0$$

Problem 2 (20 points)
If we use the initial terms of the Maclaurin series for e^x, $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$, to compute e by using $x = 1$ in e^x, through what degree must we go in order to guarantee a result within 0.0001 of the correct value of e?
(Assume that $1 < e < 3$, but that you do not know the actual value of e.)

Problem 3 (20 points)

Let $\vec{u} = 2\vec{i} - \vec{j} + 2\vec{k}$, $\vec{v} = 3\vec{i} - 4\vec{k}$, and $P = (2, -2, 3)$.

(a) Find the cosine of the angle between $\vec{u}$ and $\vec{v}$.

(b) Find the component of $\vec{u}$ in the direction of $\vec{v}$.

(c) Find the projection of $\vec{u}$ upon $\vec{v}$.

(d) Find equations in parametric form for the line through P in the direction of $\vec{v}$.

Problem 4 (18 points)
For each of the following series, tell whether it converges: If it has some negative terms, also tell whether it converges absolutely. Be sure to give reasons!

(a) $$\sum_{n=1}^{\infty}(-1)^n\left(\frac{5^n}{n!}\right)$$

(b) $$\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$$

(c) $$\sum_{n=1}^{\infty}\frac{4^n}{3^n}$$

Problem 5 (18 points)
For the points $(1, -1, 3)$, $(2, -1, 5)$, and $(1, 0, 6)$:

(a) Find a vector which is perpendicular to the plane containing the three points.

(b) Find an equation for the plane through the three points. Simplify it to the form $Ax + By + Cz = D$.

(c) Find the area of the triangle whose vertices are those three points.

Problem 6 (21 points)
Solve the differential equation

$$y'' + y = e^{2x}$$

(a) Find a particular solution to $y'' + y = e^{2x}$:

(b) Find the general solution to $y'' + y = 0$:

(c) Write the general solution to $y'' + y = e^{2x}$:

PART TWO of the final exam for Math 222, summer 2001.

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Problem 7 (22 points)
Solve (find the general solution):

$$x\frac{dy}{dx} - 3y = x^2 \qquad \text{(for } x > 0\text{)}$$

Problem 8 (20 points)
A particle moves in the plane such that its position at time t is the point (x, y) where $x = 3\sin t$ and $y = 4\cos t$, for $0 \le t \le 2\pi$.

(a) Where is the particle at time $t = 0$?

(b) Describe the motion of the particle: What is the curve in the plane along which it moves, and which direction is it going? (You may sketch a graph to aid in your description, but you should use enough words to tell what kind of curve, where it is located, the direction of motion, etc.)

Problem 9 (20 points)
Find the general solution to

$$\frac{dy}{dx} = (1 + y^2)e^x.$$

Problem 10 (20 points)
Find the first five terms (through the fourth degree term) of the Taylor series for $f(x) = \frac{1}{1-x}$ at $a = 2$.