Mathematics 222, Lecture 2 (Wilson)

Your Name: ______________________________

Circle your TA's name:

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Final Exam 5/13/98

There are thirteen problems: One of them is on the back of this sheet.
Write your answers to the problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.
Wherever applicable, leave your answers in exact forms (using π, $\sqrt{3}$, e^2, and similar numbers) rather than using decimal approximations.
You may refer to notes you have brought in on sheets of paper (regular notebook or typing size) as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	18	
2	13	
3	16	
4	12	
5	17	
6	12	
7	16	
8	15	
9	15	
10	16	
11	16	
12	16	
13	18	
TOTAL	200	

Problem 1 (18 points)

Evaluate the definite or indefinite integrals:

(a) $$\int_0^{\frac{\pi}{2}} x\ \sin^2(x)\ dx$$

(b) $$\int \frac{\sqrt{x^2-4}}{x}\ dx$$

(c) $$\int_0^3 \frac{dx}{(x-1)^{\frac{2}{3}}}$$

Problem 2 (13 points)
Use Simpson's rule with $n = 4$ to approximate

$$\int_1^3 e^x \, dx.$$

(Since this is an "easy" integral to evaluate using the fundamental theorem of calculus, credit will be given for showing how to use Simpson's rule and not just for the numerical answer! You are welcome to leave your answer as a sum of terms with numbers like e^1: If you substitute decimal values instead, please include at least four decimal places.)

Problem 3 (16 points)
Solve the initial value problem:

$$\frac{dy}{dx} = e^{2x-y}, \qquad y(0) = 1$$

Problem 4 (12 points)

(a) Show that, for any constant C,

$$y = e^{2x}\cos(x^2) + C$$

is a solution of the differential equation

$$\frac{dy}{dx} + 2e^{2x}(x\ \sin(x^2) - \cos(x^2)) = 0$$

.

(b) Find the solution of

$$\frac{dy}{dx} + 2e^{2x}(x\ \sin(x^2) - \cos(x^2)) = 0$$

which satisfies $y(0) = 0$.

Problem 5 (17 points)
Find all solutions of the differential equation

$$\frac{dy}{dx} + \frac{y}{2x+1} = \frac{\cos(x)}{\sqrt{2x+1}}$$

(Assume $x > 0$.)

Problem 6 (12 points)
A particle moves along the ellipse $4x^2 + 9y^2 = 36$. It starts at the point $(3, 0)$ at time $t = 0$. As t increases from 0 to π, the particle moves once around the ellipse in a counterclockwise direction: it returns to $(3, 0)$ when $t = \pi$. Find a parametric representation of the position of the particle at time t.

Problem 7 (16 points)
Find the area which is inside $r = 3\cos(\theta)$ but outside $r = 2 - \cos(\theta)$. Be sure to show how you find the limits of integration and what integrals you evaluate to find the area, not just a final number.

Problem 8 (15 points)
For each series, tell whether it converges or diverges, and give a reason. The reason should be based on one of the properties of series which we have studied, not just that you have used a calculator to sum a lot of terms and think you see a pattern.

(a) $$\sum_{n=1}^{\infty} \frac{n^3}{5^n}$$

(b) $$\sum_{n=1}^{\infty} \frac{2}{\sqrt{4n^2 - 2}}$$

(c) $$\sum_{n=1}^{\infty} (-1)^n \frac{2}{\sqrt{4n^2 - 2}}$$

Problem 9 (15 points)
Find the radius and interval of convergence of:

(a) $$\sum_{n=1}^{\infty} n \; 4^n \; x^n$$

(b) $$\sum_{n=1}^{\infty} \frac{(n-1)! \; x^n}{2n}$$

(c) $$\sum_{n=1}^{\infty} \frac{x^n}{(2n)!}$$

Problem 10 (16 points)
Find the Maclaurin series for $f(x) = \sin(2x)$. Be sure to show what the n^{th} term will be in general, not just a few terms from which we are expected to infer a pattern.

Problem 11 (16 points)

(a) Set up the Taylor polynomial with $n = 3$ at $a = \frac{\pi}{4}$ to approximate $\tan(\frac{\pi}{5})$. You should show how each number in the polynomial is computed, including where the value $\frac{\pi}{5}$ goes, but you do not need to evaluate the trigonometric functions.

(b) Use Taylor's formula to estimate the accuracy of your approximation in (a).

Problem 12 (16 points)
Let $\vec{a} = 4\vec{\imath} - \vec{\jmath} + 8\vec{k}$, $\vec{b} = -2\vec{\imath} - 3\vec{\jmath} + 2\vec{k}$, $\vec{c} = 2\vec{\imath} + \vec{\jmath} - \vec{k}$, $\vec{d} = 2\vec{\imath} - \vec{k}$, and $\vec{e} = 2\vec{\imath} - \frac{1}{2}\vec{\jmath} + 4\vec{k}$.

(a) Find the scalar projection of $\vec{b}$ onto $\vec{a}$.

(b) Find the vector projection of $\vec{b}$ onto $\vec{a}$.

(c) $\vec{a}$ is orthogonal (perpendicular) to exactly one of the other vectors listed: Which one?

(d) $\vec{a}$ is parallel to exactly one of the other vectors listed: Which one?

Problem 13 (18 points)
Let $P_0 = (1, 2, 3)$, $P_1 = (3, 6, 3)$, and $P_2 = (-2, 3, 9)$.

(a) Find an equation for the plane containing the points P_0, P_1, and P_2.

(b) Find the area of the triangle with vertices P_0, P_1, and P_2.