

# Mathematics 222, Lecture 2 (Wilson)

Your Name: \_\_\_\_\_

Circle your TA's name:

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Exam I      2/25/98

Write your answers to the nine problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using  $\pi$ ,  $\sqrt{3}$ , and similar numbers) rather than using decimal approximations. There is scratch paper on the back of this sheet. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on one sheet of paper (regular notebook or typing size) as announced in class.

**BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.**

Problem	Points	Score
1	11	
2	10	
3	12	
4	10	
5	12	
6	12	
7	12	
8	11	
9	10	
<b>TOTAL</b>	<b>100</b>	

# SCRATCH PAPER

**Problem 1 (11 points)**

Set up and evaluate an integral to compute the length of the curve  $y = \sqrt{4 - x^2}$  for  $0 \leq x \leq 2$ . (You might want to use a formula from geometry to check your answer.)

**Problem 2 (10 points)**

Use partial fractions to integrate

$$\int \frac{2x + 14}{(x + 2)(x - 3)} dx.$$

**Problem 3 (12 points)**

**Evaluate the integrals:**

(a)  $\int_1^2 x^2 e^{-x} dx$

(b)  $\int (25 - x^2)^{3/2} dx$

**Problem 4 (10 points)**

A circular plate of metal is standing on edge on the bottom of a tank of water. The plate is 10 feet in diameter, and the center of the plate is 25 feet below the surface of the water. Set up but do not evaluate an integral to compute the force exerted by the water on one side of the plate. (A cubic foot of water weighs about 62.5 pounds.)

**Problem 5 (12 points)**

(a) Find the general solution (i.e., a representation of all solutions) of

$$\frac{dy}{dx} = \frac{y^2 - 2y + 1}{x + 1}.$$

(b) Find the particular solution of  $\frac{dy}{dx} = \frac{y^2 - 2y + 1}{x + 1}$  which satisfies  $y(0) = 3$ .

**Problem 6 (12 points)**

**Evaluate the integrals:**

(a)  $\int x \cos(2x) dx$

(b)  $\int \cos^3(2x) \sin^2(2x) dx$

**Problem 7 (12 points)**

(a) Find the general solution (i.e., a representation of all solutions) of

$$\frac{dy}{dx} = \frac{xy + y}{x^2}.$$

(b) Find the particular solution of  $\frac{dy}{dx} = \frac{xy+y}{x^2}$  which satisfies  $y(1) = \frac{4}{e}$ .

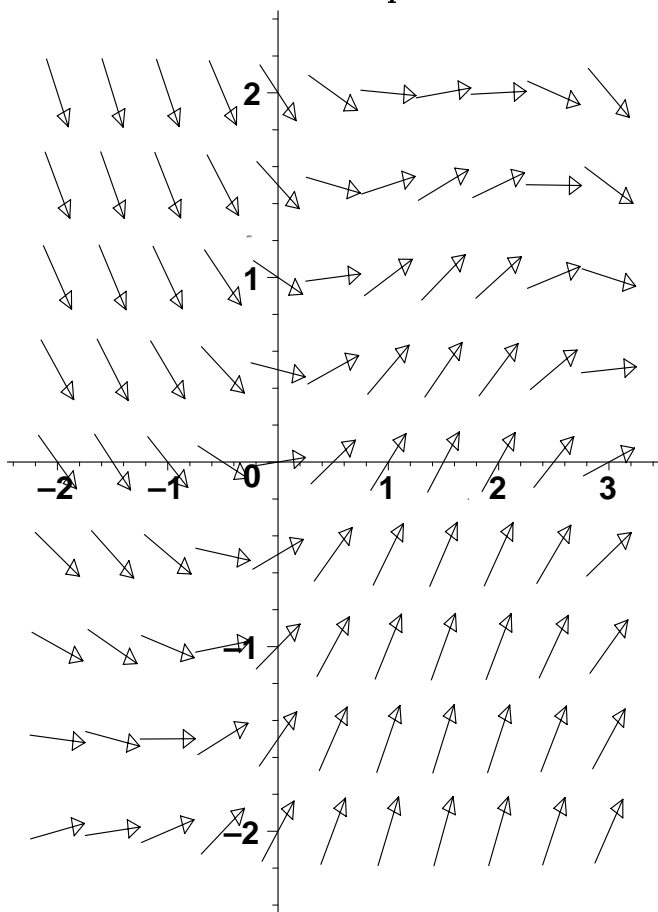
**Problem 8 (11 points)**

Find the general solution of

$$\frac{dy}{dx} + y \cos(x) = e^{-2 \sin(x)} \cos(x).$$

**Problem 9 (10 points)**

Here is a direction field plot for a differential equation  $\frac{dy}{dx} = f(x, y)$ :



(a) Draw (on the direction field above) graphs of solutions of the differential equation which satisfy (i)  $y(1) = 1$  and (ii)  $y(1) = -1$ .

(b) If  $y(1) = 1$ , approximately what will be the value of  $y(3)$ ?