

Circle your TA's name:

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Exam II 4/4/95

- Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure to tell where to look for the answer, and to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.
- You may use your calculator in doing these problems. If you do some of the work using a calculator, however, be sure to tell precisely what you asked the calculator to do. On all answers be sure to show your work.
- Unsupported answers, even if they give the correct final answer, may receive little or no credit.
- Wherever possible, (even in calculator-assisted answers!) leave your answers in exact forms (using π , e , $\sqrt{3}$, $\ln(2)$, and similar numbers) rather than using decimal approximations.
- There is scratch paper attached. If you need more scratch paper, please ask for it.
- You may refer to notes you have brought in on one sheet of paper.
- If you are asked to give a *series* as an answer, your answer must accurately describe all of the infinitely many terms. Merely writing down a few sample terms is definitely not enough. Depending on what the terms are, you might be able to write down a "generic" n^{th} term, or you might use words to describe precisely what the n^{th} term is for each n .

Problem	Points	Score
1	12	
2	11	
3	11	
4	13	
5	14	
6	13	
7	12	
8	14	
TOTAL	100	

Problem 1 (12 points)

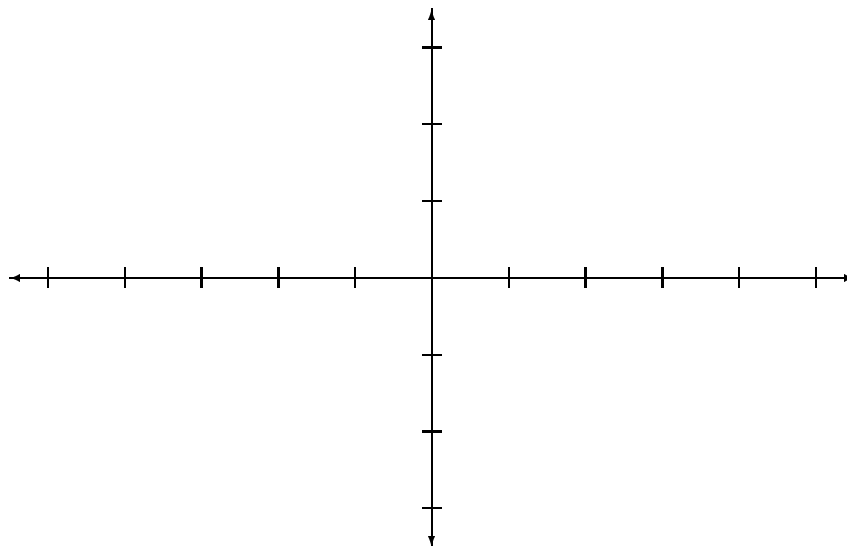
Sketch the graphs for the equations. Be sure to show or tell:

- What kind of curve is this?
- Where does the curve crosses the axes?
- What is the eccentricity of the figure graphed?

Be sure to label the axes. If it better fits your picture, you may label them so that one unit is not the same as one mark on the axes.

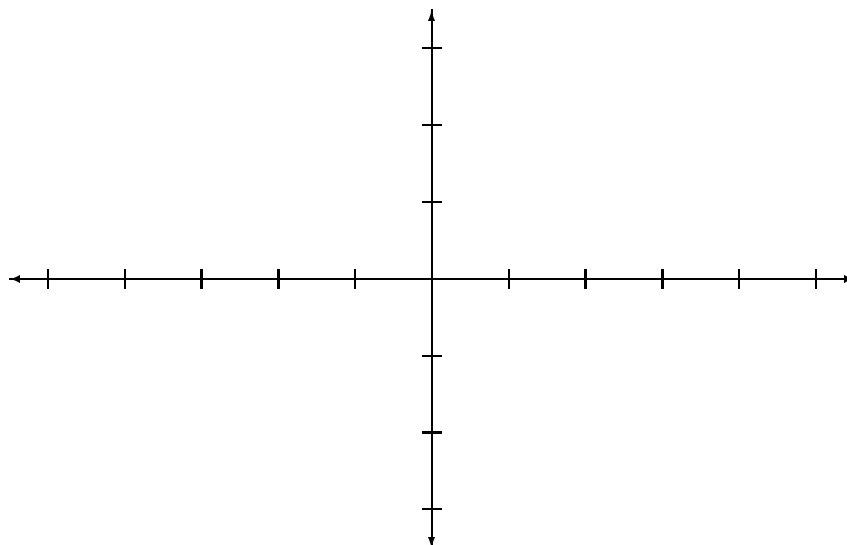
(a)

$$9x^2 - 16y^2 = 144$$



(b)

$$9x^2 + 16y^2 = 144$$



Problem 2 (11 points)

The first several terms of the Maclaurin series for $\sin(x)$ are $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$. If this expression is used to calculate an approximate value for $\sin(0.3)$, how accurate is the result? Give a bound for the error, with justification. The error bound and justification must be based on a theoretical analysis. Merely comparing the result of evaluating the expression above, for $x = 0.3$ with the result your calculator gives for $\sin(0.3)$ will not receive *any* credit.

Problem 3 (11 points)

(a) Tell whether the following series converge absolutely, conditionally, or not at all, explaining in detail why that is so:

(i)

$$\sum_{n=4}^{\infty} \frac{(-1)^n}{n \ln n}$$

(ii)

$$\sum_{n=4}^{\infty} \frac{(-1)^n}{\ln n}$$

(b) What are the *radius* and the *interval* of convergence for the series below?

$$\sum_{n=1}^{\infty} \frac{x^n \sqrt{n}}{3^n}$$

(You should check for convergence at the endpoints of the interval, but that will not be a major portion of the credit on this problem.)

Problem 4 (13 points)

(a) A particle moves along the curve $\frac{x^2}{25} - \frac{y^2}{9} = 1$.

(i) Find a parametrization of the curve which corresponds to the particle making one trip along the left portion of the curve from top to bottom.

(ii) Find a parametrization of the curve which corresponds to the particle making one trip along the right portion of the curve from bottom to top.

(b) Find an equation for the tangent line to the curve parametrized by $x = \frac{2}{t}$, $y = 2 - \ln t$, at the point where $t = 1$.

(Notice this is a new curve, not the same as in (a)!))

Problem 5 (14 points)

Find the Maclaurin series for

$$f(x) = e^x - e^{-x}$$

You should show how to compute the coefficient on x^n from the definition, not just combine series you already know.

Be sure you have read the instructions on the first page, which refer to writing an answer which includes a series.

Problem 6 (13 points)

We need to calculate the value of e^x at $x = 0.5$, and we decide to use the Maclaurin series for e^x to do the job. We need the answer accurate to within plus or minus 0.00001 , 10^{-5} . How many terms of the series do we need to use? Justify your answer.

(Your must be based on some form of Taylor's theorem with remainder: Merely evaluating different polynomials and comparing the results with the result your calculator gives for $e^{0.5}$ will not receive *any* credit.)

Problem 7 (12 points)

Set up but do not evaluate integrals to calculate the lengths of the following parametrized curves.

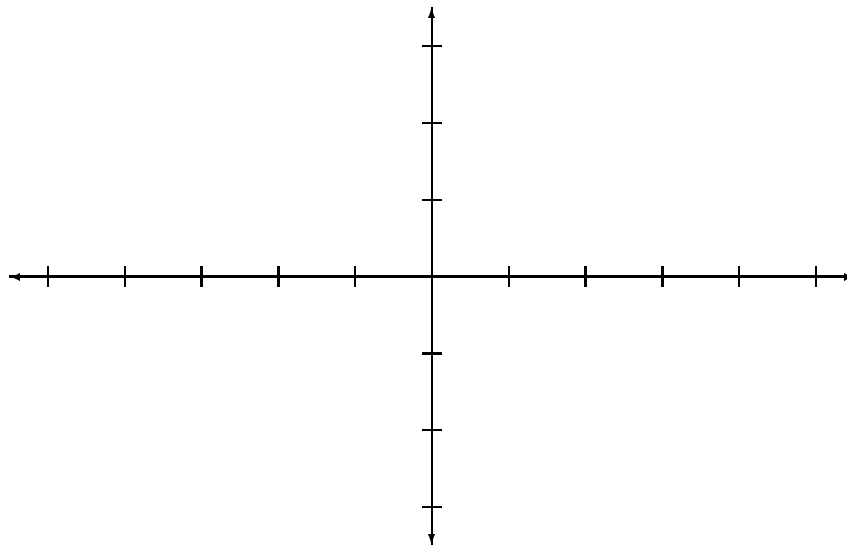
Note: Length of the curve is used as in section 9.4 of the text. That means it is the length of the path, not the distance a particle might go if it went along the path with some backing up and retracing of portions it has previously visited. The parametrization given below may not traverse the points exactly once: Check for that! You may need to revise the parametrization to visit all of the points exactly once.

(a) $x = 12 \cos t$ and $y = 5 \sin t$, for $-\pi \leq t \leq 2\pi$.

(b) $x = t^2$ and $y = 2t - 1$, for $-5 \leq t \leq 3$.

Problem 8 (14 points)

(a) Sketch the graph of the equation $r^2 = 4 \cos \theta$ in polar coordinates:



(b) Give all the polar coordinate pairs representing the point whose rectangular coordinates are $(-1, \sqrt{3})$.

SCRATCH PAPER