Your Name: ______________________________

Circle your TA's name:

Adam Berliner		**Patrick Curran**	**Sam Eckels**
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Mathematics 222, Spring 2008 **Lecture 1 (Wilson)**

Final Exam May 14, 2008

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is a problem on the back of this sheet: Be sure not to skip over it by accident!

There is scratch paper at the end of this exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought on three sheets of paper, as announced in class. You may also use the sheet on Undetermined Coefficients which was distributed in class.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. If you use a formula, state what it is. If you make a substitution, show exactly what it does and how. Do not assume the grader will try to read your mind!

Problem	Points	Score
1	30	
2	30	
3	30	
4	22	
5	20	
6	20	
7	20	
8	28	
TOTAL	200	

Problem 1 (30 points)
Evaluate the integrals:

(a) $\displaystyle\int x^2 \ln(x)dx$

(b) $\displaystyle\int_0^{\frac{\pi}{3}} \sin^3(x)\, dx$

(c) $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

Problem 2 (30 points)

(a) An object moves along the parabola $x = -4y^2$. At the instant when $t = 0$ it is at the point $(-16, 2)$. It moves through the origin, moving along the curve without backing up, and reaches the point $(-36, -3)$ at the instant when $t = 5$.

Find a parametric representation for this motion. I.e., find functions $f(t)$ and $g(t)$ such that setting $x = f(t)$ and $y = g(t)$ for $0 \leq t \leq 5$ gives position (x, y) as described above.

(b) A conic section has the following properties:

- It crosses the $y-$axis at $(0, \pm 3)$.
- It has foci at $(0, \pm 5)$.
- It is symmetric about the $y-$axis and also about the $x-$axis.
- It does not cross the $x-$axis at all.

What kind of conic section (circle, ellipse, parabola, hyperbola) is this? What is its eccentricity? Write an equation for this curve.

Problem 3 (30 points)

(a) Let $f(x) = \ln(1+x)$. Find the Maclaurin series for $f(x)$ through the term with x^3. Be sure to show how the coefficients are found, using derivatives and factorials: Do not just recite the series from memory or your notes.

(b) Let $p(x)$ be the polynomial $p(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120}$. Suppose we use this (some of the initial terms of the Maclaurin series for e^x) to approximate $e = e^1$. How accurate will this approximation be?

(Numerically, $e \approx 2.718281828$ and $p(1) \approx 2.716666667$, so we can calculate very closely just how far off the approximation is. Your job is to use the remainder term from Taylor's theorem to produce a bound for the error, not just to find the difference between these numbers.)

Problem 4 (22 points)
Find the general solution $y(x)$ to $y'' - 4y' + 4y = 2\sin(2x)$.

Problem 5 (20 points)

Let P be the point $(1,2,5)$ and Q the point $(1,1,4)$ in space. Consider the plane $2x-2y+z=4$.

(a) Find the distance from the point P to the plane $2x-2y+z=4$. (Hint: Project $\overrightarrow{PQ}$ onto a vector normal to the plane.)

(b) Write $\overrightarrow{PQ}$ as the sum of two vectors $\overrightarrow{PQ}=\vec{u}+\vec{v}$ such that $\vec{u}$ is perpendicular to the plane $2x-2y+z=4$ while $\vec{v}$ is perpendicular to $\vec{u}$.

Problem 6 (20 points)
Let P, Q, and R be the points $P = (2, 1, -3)$, $Q = (3, 1, -2)$, and $R = (2, 3, -1)$ in space.

(a) Find an equation for the plane passing through P, Q, and R.

(b) What is the area of the triangle whose vertices are P, Q, and R?

Problem 7 (20 points)
Solve the initial value problem $y' + 2xy = x$ with $y(0) = 0$.

Problem 8 (28 points)

For each of the following series, tell which (one or more) of these terms applies: (i) Diverges (ii) Converges (iii) Converges Conditionally (iv) Converges Absolutely. Be sure to give reasons!

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(2n)!}{n!\,n!}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

(c) $\displaystyle\sum_{n=0}^{\infty} x^n$, if $-1 < x < 1$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4+1}$

Scratch Paper

(not a command!)