Your Name: ______________________________

Circle your TA's name:

Adam Berliner		**Patrick Curran**	**Sam Eckels**
Zheng Hua	**James Hunter**	**Jason Murcko**	**Dan Rosendorf**

Mathematics 222, Spring 2008 **Lecture 1 (Wilson)**

Second Midterm Exam April 24, 2008

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is a problem on the back of this sheet: Be sure not to skip over it by accident!

There is scratch paper at the end of this exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought on one sheet of paper or two 6x8 index cards, as announced in class and by email. You may also use the sheet on Undetermined Coefficients which was distributed in class.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. If you use a formula, state what it is. If you make a substitution, show exactly what it does and how. Do not assume the grader will try to read your mind!

Problem	Points	Score
1	12	
2	13	
3	12	
4	13	
5	12	
6	13	
7	13	
8	12	
TOTAL	100	

Problem 1 (12 points)
For any given number x the formula $a_n = x^n$ for $n = 1,\ 2, \ldots$ defines numbers $a_1,\ a_2, \ldots$ Using that formula:

(a) For what choices of x does the sequence $\{a_n\}$ converge?
When it converges, to what does it converge?
Explain your reasoning.
(You do not have to give a proof using ϵ.)

(b) For what choices of x does the series $\sum_{n=1}^{\infty} a_n$ converge?
When it converges, to what does it converge?
Explain your reasoning.
(For those x-values where you say it converges, and where you say it diverges, be sure to explain how you can be sure.)

Problem 2 (13 points)

Solve the initial value problem: $$e^x \frac{dy}{dx} + e^x\, y = \sin x \qquad \text{AND} \qquad y(0) = 0.$$

Be sure to indicate clearly what function is your final answer.

Problem 3 (12 points)
For each series, tell whether it converges Absolutely, Conditionally, or Not at All: Be sure to give reasons for your answers, citing the convergence tests you used. Make sure you point out how the requirements of a test are satisfied.

(a) $\sum_{n=2}^{\infty}\left[(-1)^{n+1}\ \frac{\sin(2n)+2}{n-1}\right]$

NOTE: This part of the problem was changed at the exam to "This series does converge: Does it converge absolutely or conditionally?" See the online answers to find out why...

(b) $\sum_{n=1}^{\infty}(-1)^{n}\ \frac{2}{n^{\sqrt{2}}}$

(c) $\sum_{n=1}^{\infty}\ \frac{1}{n}$

Problem 4 (13 points)

(a) Let $f(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1}\, \frac{x^n}{n} + \cdots$, wherever that series converges.

(i) Find the interval of convergence for this power series. Be sure to check for (absolute or conditional) convergence at each end of the interval, if it is not the set of all real numbers or just $\{0\}$.

(ii) (3 extra credit points) If x is a number for which this series converges absolutely, what is $f(x)$?

(I.e., express $f(x)$ in terms of trigonometric functions, logarithms, exponential functions, polynomials, and similar functions not written as series.)

(b) Using the formula for coefficients that involves derivatives, find the Maclaurin series for $f(x) = \sin x + \cos x$. (This is not the same f as in part (a).) Show your work,

Do not just add the series for $\sin x$ and $\cos x$ except perhaps as a check on your answer.

Either write a formula for the n^{th} term in general or write out all terms through the one involving x^8.

Problem 5 (12 points)

(a) For $f(x) = 3 + x - 2x^2 + 4x^3$: Find the Taylor series for $f(x)$ at $a = 1$.

Write out all the non-zero terms or, if there are infinitely many, describe a pattern that they follow.

(b) We know that the area in a circle of radius r is given by the formula $A = \pi r^2$.

Set up and evaluate an integral in polar coordinates to derive that formula.

Problem 6 (13 points)
Find the general solution of

$$y'' + 2y' - 3y = -30 \sin 3x.$$

Be sure to indicate which function is your final answer.

Problem 7 (13 points)

Suppose we use $x - \frac{x^3}{6} + \frac{x^5}{120}$ (the first few terms of the Maclaurin series for $\sin(x)$) as an approximation to $\sin(x)$, for $0 \leq x \leq \frac{1}{2}$.

(a) What is the maximum possible error, i.e. how much could $\sin(x)$ differ from $x - \frac{x^3}{6} + \frac{x^5}{120}$ on that range of x-values?

Be sure to show in detail how you got your error bound. Do not just write down some numbers. In particular, using a calculator to find values of $x - \frac{x^3}{6} + \frac{x^5}{120}$ and of $\sin(x)$ and simply comparing the results will get no credit.

(b) Will the value $x - \frac{x^3}{6} + \frac{x^5}{120}$ be larger or smaller than the actual value of $\sin(x)$ for those x-values?

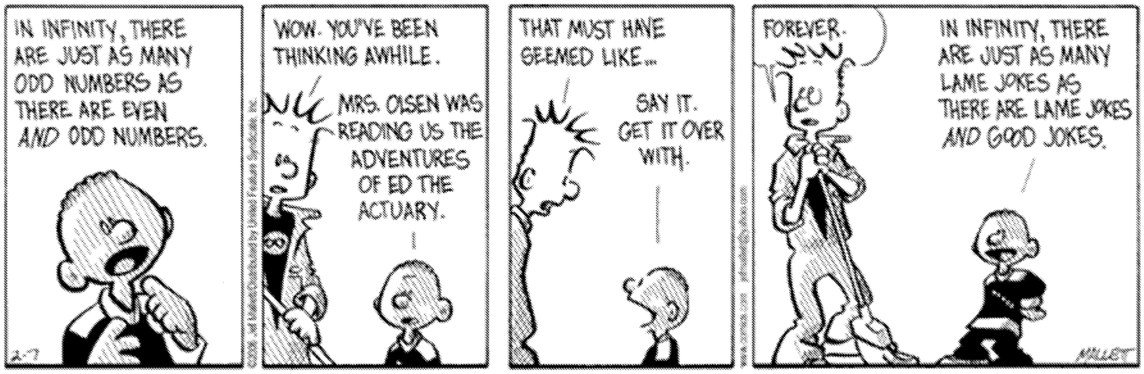
Again, show how you get your answer and why it is correct.

Problem 8 (12 points)
For each series, tell whether it converges or diverges. Be sure to give a reason based on one of the convergence tests we have studied. If you make algebraic changes in the terms be sure to show what you have done, e.g., to justify a comparison.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3n}{n^3+2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{n^2+5}$



Scratch Paper

(not a command!)