Your Name: ____

Circle your TA's name:

Adam Berliner		**Patrick Curran**	**Sam Eckels**
Zheng Hua	**James Hunter**	**Jason Murcko**	**Dan Rosendorf**

Mathematics 222, Spring 2008 **Lecture 1 (Wilson)**

First Midterm Exam February 28, 2008

Write your answers to the seven problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is scratch paper at the end of this exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought on one half sheet of paper or a 6x8 index card, as announced in class and by email.
There are also some formulas given on the other side of this sheet.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)

Problem	Points	Score
1	18	
2	18	
3	18	
4	10	
5	10	
6	12	
7	14	
TOTAL	100	

Some formulas, identities, and numeric values you might find useful:

Values of trig functions:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	—

Trig facts:

1. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
2. $\sec\theta = \frac{1}{\cos\theta}$
3. $\sin^2\theta + \cos^2\theta = 1$
4. $\sec^2\theta = \tan^2\theta + 1$
5. $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
6. $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
7. $\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$
8. $\sin^2 x = \frac{1}{2}(1-\cos 2x)$
9. $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

Derivative formulas:

1. $\frac{d}{dx}\tan x = \sec^2 x$
2. $\frac{d}{dx}\sec x = \sec x\,\tan x$
3. $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
4. $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$
5. $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$
6. $\frac{d}{dx}\ln x = \frac{1}{x}$
7. $\frac{d}{dx}e^x = e^x$

Integral formulas:

1. $\int u^n\,du = \frac{1}{n+1}u^{n+1} + C$, if $n \neq -1$
2. $\int \frac{1}{u}\,du = \ln|u| + C$
3. $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$
4. $\int \frac{du}{1+u^2} = \tan^{-1} u + C$
5. $\int \sec(u)\,du = \ln|\sec(u)+\tan(u)| + C$
6. $\int u\,dv = uv - \int v\,du$

Algebra formulas:

1. $\ln(xy) = \ln(x) + \ln(y)$
2. $a^{x+y} = a^x\,a^y$
3. $a^b = e^{b\ln a}$

Problem 1 (18 points)

Evaluate the integrals:

(a) $\int e^{2x} \sin(x)\, dx.$

(b) $\int \sec^4(x)\, dx.$

Problem 2 (18 points)

(a) Evaluate $\displaystyle\int_0^2 \frac{2x\,dx}{\sqrt{4-x^2}}$.

(b) Five measured values of a function $f(x)$ are given in the table at the right. Use the trapezoidal rule to approximate $\displaystyle\int_3^5 f(x)\,dx$. You do not need to calculate an error estimate.

x	$f(x)$
3	2
3.5	1
4	3
4.5	4
5	6

Problem 3 (18 points)

(a) Evaluate $\displaystyle\int_0^{\frac{1}{2\sqrt{2}}} \frac{2\,dx}{\sqrt{1-4x^2}}$.

(b) Evaluate $\displaystyle\int x^2 \ln(x)\,dx$.

Problem 4 (10 points)

A hyperbola in standard position, its center at the origin and its foci on one of the coordinate axes, passes through the (rectangular coordinates) points $(0, \pm 12)$, and it is asymptotic to the lines $y = \pm \frac{12}{5} x$.

(a) Find an equation for this hyperbola.

(b) Where are the foci of his hyperbola? (Give specific coordinates.)

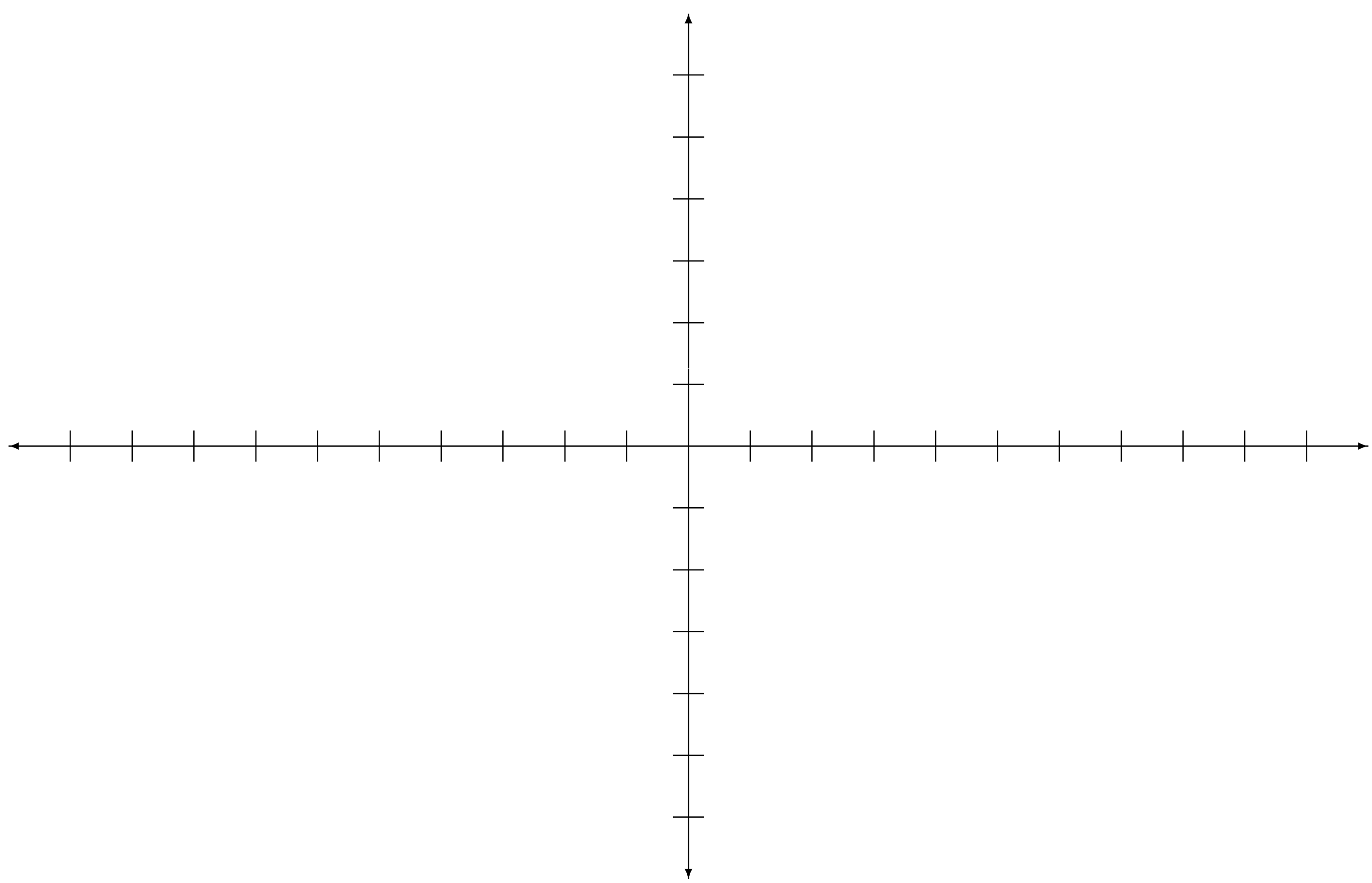
(c) What is the eccentricity of this hyperbola?

Problem 5 (10 points)

A curve is described by the equation $\frac{x^2}{100} + \frac{y^2}{36} = 1$.

(a) What kind of conic section is this curve?

(b) Graph the curve: You should label and give specific coordinates for any places where it crosses the $x-$ and $y-$axes, foci, etc. Your graph must not have sharp corners where it should really be smooth, but otherwise you won't be graded on artistic ability. I have provided marks along the axes to help you, but it is up to you to decide what lengths go with the marks: Label the marks to indicate where you have located one unit along each axis.



(c) What is the eccentricity of this curve?

Problem 6 (12 points)
An object is intended to move so that its position at time t, for $0 \le t \le 2\pi$, will be given by $x = 3\cos t$ and $y = 5\sin t$.

(a) Describe this motion using words and/or an equation in rectangular coordinates. Be sure to tell how far up/down and how far left/right the object would move and to tell what kind of conic section it follows. Tell where it starts, what direction it moves, and where it finishes.

(b) At the instant when $t = \frac{\pi}{4}$, the forces holding the object on that curve fail and the object flies away along the tangent line to the path it had been on.

Find an equation for that tangent line.

Problem 7 (14 points)

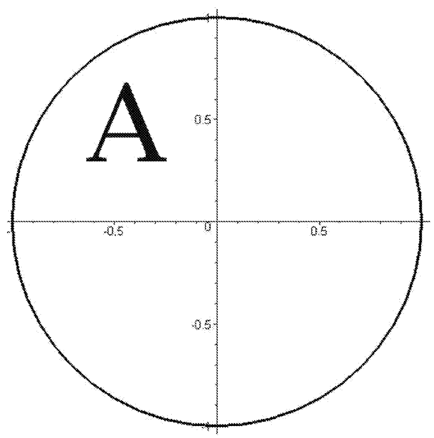
Consider two curves given in polar coordinates:

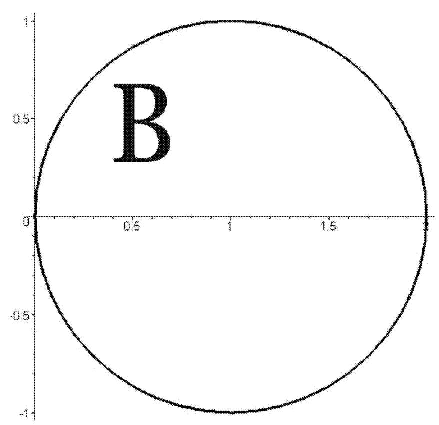
(curve (i)) $r = 1.$

(curve (ii)) $r = 2\cos\theta.$

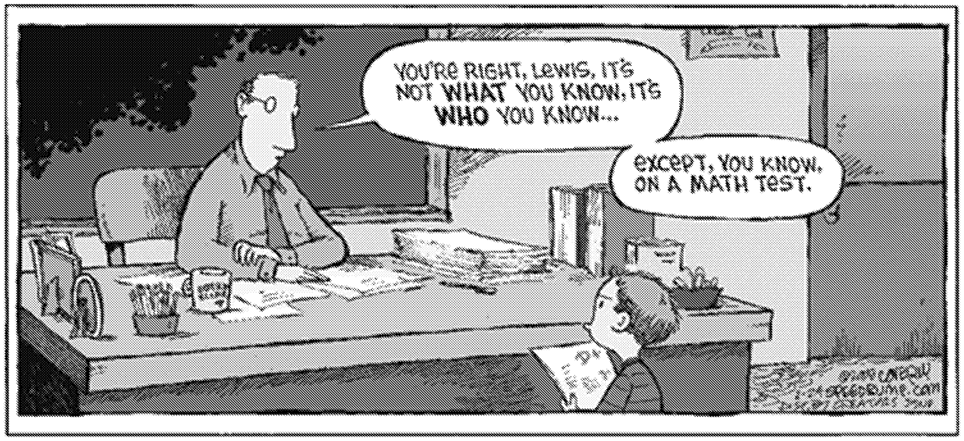
(a) Find the point(s) where these two curves intersect. Give both the polar and the rectangular coordinates of each intersection point, being sure to label which coordinates are polar and which are rectangular.

(b) At the right are two graphs. Which one (A or B) is curve (i), and which one is curve (ii)?





(c) One of the intersection points (and only one) is in the first quadrant, where $x \geq 0$ and $y \geq 0$. At that point, find the slope and the tangent line (equation in rectangular coordinates) to curve (ii).



Scratch Paper

(not a command!)