Your Name: _____

Circle your TA's name:

Adam Berliner John Bowman Chris Holden Eugene Tsai Dan Turetsky

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Mathematics 222, Spring 2005 Lecture 4 (Wilson)

Second Midterm Exam April 12, 2005

Write your answers to the nine problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

You may refer to notes you have brought on two index cards, as announced in class and on the class website.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)

Problem	Points	Score
1	11	
2	11	
3	11	
4	11	
5	12	
6	12	
7	10	
8	12	
9	10	
TOTAL	100	

Problem 1 (11 points)

The series $\sum_{n=0}^{\infty}(-1)^n \frac{2n}{3^n+1}$ does converge, to some number L.

(a) Give a reason, based on some theorem we have studied, that assures us this series really does converge. Be sure to show why the theorem applies.

(b) If we add up only the terms of this series corresponding to $n = 0$ through $n = 9$, how far can the resulting sum differ from L?

(c) If we use the sum from (b), just the first ten terms, will that sum be larger than L or will it be smaller than L? Be sure to give a reason for your answer!

Problem 2 (11 points)

Find the area of the region which is inside the circle $r = 1$ but outside the cardioid $r = 1+\cos\theta$. Be sure to show how you find the limits of integration: Explain what (if any) symmetries you are using, show how you find particular θ values that matter, etc. A sketch may be helpful, but you should not just read numbers from a picture.

Problem 3 (11 points)

An ellipse has one focus at $(0, 4)$ and the other focus at $(12, 4)$, and the length of the major axis (the length from one end of the ellipse to the other end in the longest direction) is 20.

Find an equation for this ellipse.

(For this problem all coordinates are rectangular, (x, y), coordinates, not polar coordinates.)

Problem 4 (11 points)

For $f(x) = (x+1)^{3/2}$, find the first few terms of the Taylor series for $f(x)$ at $a = 8$.

Explicitly write the series through (including) the term of degree 3. You do not have to construct an expression for the n^{th} term in general.

Problem 5 (12 points)

(a) Find the solution of $y'' - 6y' + 9y = 0$ that satisfies $y(0) = 1$ and $y'(0) = 1$. (Your answer should be a specific function $y = f(x)$ with no unevaluated constants.)

(b) For the different differential equation $y'' + 6y' + 13y = 0$:

Find an expression for all solutions $y(x)$.

Problem 6 (12 points)

For each of the following three series, tell whether it converges Absolutely, Conditionally, or Not At All. Be sure to give reasons for your answers!

(a) $\displaystyle\sum_{n=3}^{\infty}(-1)^n\frac{2n}{3n+\cos(n)}$

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{\sin(n+1)}{n^2}$

(c) $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots$

Problem 7 (10 points)

A function $f(x)$ takes on the values given in the table:

x	$f(x)$
3	0
5	5
7	14
9	27
11	44

(a) Use the Trapezoidal Rule to compute an approximation to $\int_3^{11} f(x)\,dx$. You do not need to estimate error.

(b) Use the Parabolic Rule (Simpson's Rule) to compute an approximation to $\int_3^{11} f(x)\,dx$. You do not need to estimate error.

Problem 8 (12 points)

Suppose we choose to use the terms of the Maclaurin series for e^x through $\frac{x^4}{4!}$ to approximate the true value of e^x. If we only apply this to x-values between -2 and -1, how accurate will our results be?

You should come up with an expression for a number (It does not have to be written as a decimal fraction!) such that you can be sure the actual error for any $x \in (-2, -1)$ is at most that number. You must provide an explanation showing why your number is guaranteed to work.

Problem 9 (10 points)

(a) Find the radius of convergence and the interval of convergence for the series $\sum_{n=1}^{\infty}(-1)^n \frac{(2x)^n}{n\,3^n}$. Be sure to show your reasoning.

(b) Does the series $\sum_{n=0}^{\infty} \frac{3^{n+1}}{5^n}$ converge? If your answer is no, tell how you know that. If your answer is yes, tell what the series converges to, i.e.: What is the sum of the series?