Mathematics 222, Lecture 2 (Wilson)   Your Name: ______________________________

Exam 1   February 21, 2002

- **This is the portion of the exam for just our section. You should also have an exam portion which is common to all the small 222 sections this semester.**
- **Write your answers to the three problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.**
- **Wherever applicable, leave your answers in exact forms (using π, e, $\sqrt{3}$, $\ln(2)$, and similar numbers) rather than using decimal approximations.**
- **You may refer to notes you have brought in on one sheet of paper or two index cards, as announced in class.**

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	12	
2	12	
3	12	
TOTAL	36	

Problem 1 (12 points)

(a-1) Find an expression for the sum s_n of the first n terms of the series

$$\frac{2}{2\cdot 3}+\frac{2}{3\cdot 4}+\frac{2}{4\cdot 5}+\cdots=\sum_{n=2}^{\infty}\frac{2}{n(n+1)}.$$

(a-2) What is the sum of this series?

(b) The series

$$\sum_{n=1}^{\infty}(-1)^n\,\frac{3}{2n+1}$$

does converge. If we compute the sum of just the first 5 terms:
(i) How much could this sum differ from the sum of the entire series?
(ii) Would the sum of the first 5 terms be larger or smaller than the sum of the series?

Problem 2 (12 points)

For each of these series, tell whether it converges. Regardless of whether your answer is "converges" or "diverges," be sure to give reasons for your answer!

(a) $$\sum_{n=1}^{\infty} \frac{n^5}{5^n}$$

(b) $$\sum_{n=1}^{\infty} \frac{1}{n^3+2}$$

(c) $$\sum_{n=1}^{\infty} \frac{2}{n+\ln n}$$

Problem 3 (12 points)
For each of these series, tell whether it converges absolutely, conditionally or not at all. No matter what the rest of your answer, be sure to give reasons!

(a) $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{2n^2 - 1}$$

(b) $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{2n - 1}$$

(c) $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin(n)}{n^2}$$