Mathematics 222, Lecture 1 (Wilson)

Your Name: ______________________________

Circle your TA's name:

Gautam Bharali　　Jeremy Hoffmann　　Prabu Ravindran

Jamie Sutherland　　Nat Thiem

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Write your answers to the 12 problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using π, $\sqrt{3}$, and similar numbers) rather than using decimal approximations. There is scratch paper on the back of this sheet. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on three index cards, as announced in class. You may use a calculator, but remember:

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	18	
2	18	
3	16	
4	16	
5	15	
6	16	
7	18	
8	18	
9	16	
10	16	
11	16	
12	17	
TOTAL	200	

SCRATCH PAPER

Problem 1 (18 points)

Evaluate the integrals:

(a) $$\int x \sin(\frac{x}{2})\, dx$$

(b) $$\int \frac{dx}{x^2\sqrt{x^2-1}}$$

(You may assume $x > 1$.)

Problem 2 (18 points)

(a) Find the terms of the Taylor series for $f(x) = \ln(x+2)$ at $a = 1$ through the 5^{th} degree term. For a possible 3 extra points, write a formula for the n^{th} degree term in general.

(b) Find the Maclaurin series for $f(x) = e^x + \cos(2x)$. You should include a formula for the n^{th} degree term in general.

Problem 3 (16 points)

Find all solutions of the differential equation

$$e^x \frac{dy}{dx} + 2e^x y = 1.$$

(Hint: You can convert this to the standard form for first order linear equations.)

Problem 4 (16 points)

The formula

$$1 - \frac{4x^2}{2} + \frac{16x^4}{24}$$

is used to approximate $\cos(2x)$ on the interval $-1 < x < 1$. Give and justify a bound on how big the error arising in this approximation can be.

Problem 5 (15 points)

Find all solutions of the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 5y = 0.$$

Problem 6 (16 points)

(a) Find an equation for the plane through the points P $(1, -2, 3)$, Q $(1, 0, 2)$, and R $(2, -2, 4)$.

(b) Is the plane $-4x + 2y + 4z = 17$ parallel to the plane you got in (a)?

Problem 7 (18 points)

(a) Find an equation for the hyperbola which has vertices at $(0, \pm 2)$ and has the lines $y = \pm x$ as asymptotes.

(b) A conic section has equation $9y^2 = 54 - 6x^2$. Identify the kind of conic, tell where it crosses the axis or axes, and give its eccentricity.

Problem 8 (18 points)

Let $\vec{a} = 2\vec{i} - 2\vec{j} - \vec{k}$ and $\vec{b} = 3\vec{i} + 4\vec{k}$.

(a) What is the angle between the vectors $\vec{a}$ and $\vec{b}$? (You may express the angle using an inverse trig function.)

(b) Find the vector projection of $\vec{b}$ onto $\vec{a}$.

(c) Find a (non-zero) vector which is perpendicular both to $\vec{a}$ and to $\vec{b}$.

Problem 9 (16 points)

Find an equation for the tangent line to the hyperbola $x = \sec(t)$, $y = \tan(t)$, at the point $(\sqrt{2}, 1)$ where $t = \frac{\pi}{4}$.

Problem 10 (16 points)

Let P be the point $(1, 3, 5)$ and Q the point $(2, 1, 6)$.

(a) Find equations in both parametric and symmetric forms for the line through P and Q.

(b) Find an equation for the plane through P which is perpendicular to the line found in (a).

Problem 11 (16 points)

Find the area of the region which is inside both the circle $r = 2\sin(\theta)$ and the circle $r = 2\cos(\theta)$.

Problem 12 (17 points)

Find the solution to the initial value problem

$$2y\,\frac{dy}{dx} = \frac{x^2+1}{\cos(y^2)} \qquad \text{with} \qquad y(0) = \sqrt{\frac{\pi}{2}}.$$