

Your Name: _____

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Mathematics 222, Spring 2007

Lecture 3 (Wilson)

Final Exam May 17, 2007

There is a problem on the back of this sheet! Do not accidentally skip over it!

Write your answers to the ten problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever possible, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. For example, $\sin(\frac{\pi}{6}) = \frac{1}{2}$, and writing something like .499 may not get you full credit. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is scratch paper at the end of this exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought on up to three sheets of paper, and the class handout on undetermined coefficients, as announced in class and by email.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. (“I did it on my calculator” and “I used a formula from the book” (without more details) are not sufficient substantiation...)

Problem	Points	Score
1	15	
2	20	
3	20	
4	21	
5	20	
6	21	
7	20	
8	21	
9	21	
10	21	
TOTAL	200	

Problem 1 (15 points)

For each of the following equations, indicate its graph by filling in the blank with the number from the picture below.

(a) _____ $r = 1 + \sin \theta$

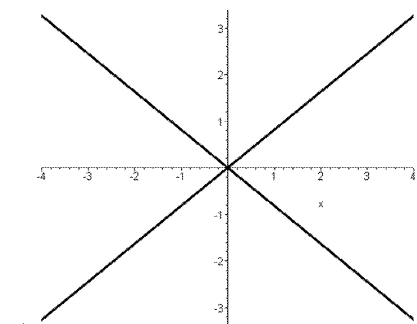
(d) _____ $\frac{x^2}{9} - \frac{y^2}{4} = 0$

(b) _____ $-\frac{x^2}{9} + \frac{y^2}{4} = 1$

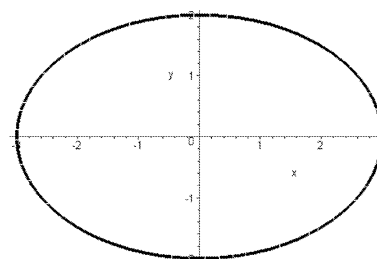
(e) _____ $\frac{x^2}{9} + \frac{y^2}{4} = 1$

(c) _____ $r = 2 \cos(4\theta)$

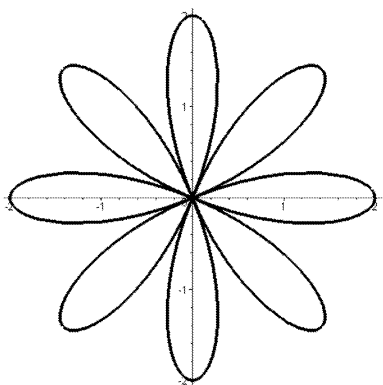
(f) _____ $x = -\frac{1}{4}y^2$



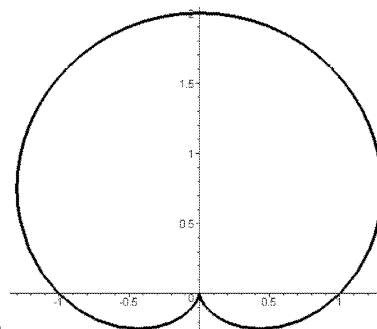
1



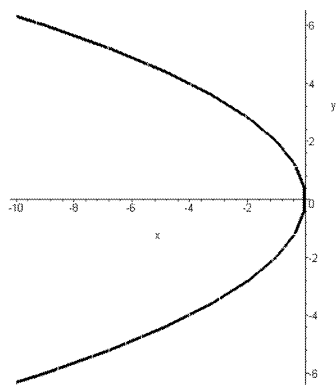
2



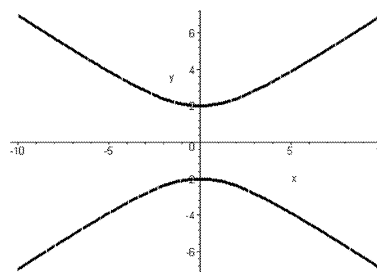
3



4



5



6

Problem 2 (20 points)

Evaluate the integrals:

(a) $\int x^2 \cos(x) dx$

(b) $\int \tan^3(x) dx$

Problem 3 (20 points)

Evaluate the integrals:

(a)
$$\int_0^{\frac{2}{3}} \sqrt{4 - 9x^2} dx$$

(b)
$$\int_{-8}^{27} x^{-4} dx$$

Problem 4 (21 points)

For each of the following series, tell whether it converges or diverges. If it converges and it has some positive and some negative terms, tell whether it converges absolutely or conditionally. Be sure to give reasons justifying your answers.

(a)
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2 + 1}$$

(b)
$$\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{n-1}$$

(c)
$$\sum_{n=1}^{\infty} n e^{-n^2}$$

Problem 5 (20 points)

Find the Taylor series at $a = 1$ for $f(x) = \cos(x^2 - 1)$. Show the terms through the 3rd degree term.

Derive the coefficients from the general form for Taylor series, do not just “plug in” to some known series. That is, you should calculate a_0 , a_1 , a_2 , and a_3 using derivatives, and show how they are fitted into a third-degree polynomial.

Problem 6 (21 points)

Find all solutions of the differential equation

$$y'' + 2y' + y = 6 \sin(2x).$$

Problem 7 (20 points)

Let $\vec{u} = 2\vec{i} + \vec{j} + \vec{k}$ and $\vec{v} = \vec{i} - 2\vec{j} + 2\vec{k}$.

- (a) What is $|\vec{v}|$, the magnitude of \vec{v} ?
- (b) What is the $\cos \theta$, if θ is the angle between \vec{u} and \vec{v} ?
- (c) What is the scalar component of \vec{u} in the direction of \vec{v} ?
- (d) What is $\text{proj}_{\vec{v}}\vec{u}$, the projection of \vec{u} on \vec{v} ?
- (e) Find two vectors \vec{u}_1 and \vec{u}_2 such that (i) $\vec{u} = \vec{u}_1 + \vec{u}_2$, (ii) \vec{u}_1 is parallel to \vec{v} , and (iii) \vec{u}_2 is orthogonal to \vec{v} .

Problem 8 (21 points)

Suppose the polynomial $1 - \frac{x^2}{2} + \frac{x^4}{24}$ is used to calculate, approximately, $\cos(x)$. If this will be used for values of x from -1 to 1 , what accuracy can you guarantee will be achieved?

Your answer should use the remainder term from Taylor's theorem in showing your answer is valid. If you know another mathematically correct way to do the problem you can use that as a check on your answer and get up to 5 extra points. But it will not substitute for an answer using Taylor's theorem.

Problem 9 (21 points)

Solve the initial value problem

$$x \frac{dy}{dx} + 2y = x^3 \quad (\text{for } x > 0) \quad \text{and} \quad y(2) = 1.$$

Problem 10 (21 points)

Consider two planes, Π_1 and Π_2 , given by

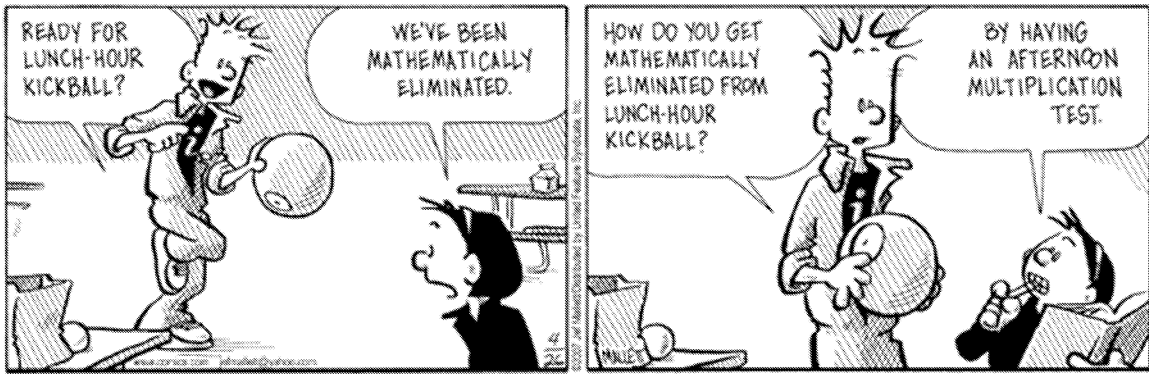
$$\Pi_1: \quad x + 2y - z = 7$$

and

$$\Pi_2: \quad 2x + 3y + 2z = 4.$$

- (a) Find parametric equations for the line of intersection of these two planes.
Hint: The point $(1, 2, -2)$ is on both planes.

- (b) The angle between two planes means the angle between a vector perpendicular to one plane and a vector perpendicular to the other. What is the cosine of the angle between Π_1 and Π_2 ?



Scratch Paper

(not a command!)