Your Name: ________________________________

Circle your TA's name:

Lino Amorim	**Jon Godshall**	**Ed Hanson**
Elizabeth Mihalek	**Rob Owen**	**Kim Schattner**

Mathematics 222, Spring 2007 **Lecture 3 (Wilson)**

First Midterm Exam March 1, 2007

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is scratch paper at the end of this exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought on one sheet of paper, as announced in class and by email.
There are also some formulas given on the other side of this sheet.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)

Problem	Points	Score
1	16	
2	10	
3	10	
4	11	
5	11	
6	14	
7	12	
8	16	
TOTAL	100	

Some formulas, identities, and numeric values you might find useful:

Values of trig functions:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	—

Trig facts:

1. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
2. $\sec\theta = \frac{1}{\cos\theta}$
3. $\sin^2\theta + \cos^2\theta = 1$
4. $\sec^2\theta = \tan^2\theta + 1$
5. $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
6. $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
7. $\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$
8. $\sin^2 x = \frac{1}{2}(1-\cos 2x)$
9. $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

Derivative formulas:

1. $\frac{d}{dx}\tan x = \sec^2 x$
2. $\frac{d}{dx}\sec x = \sec x\,\tan x$
3. $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
4. $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$
5. $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$
6. $\frac{d}{dx}\ln x = \frac{1}{x}$
7. $\frac{d}{dx}e^x = e^x$

Integral formulas:

1. $\int u^n\,du = \frac{1}{n+1}u^{n+1} + C$, if $n \neq -1$
2. $\int \frac{1}{u}\,du = \ln|u| + C$
3. $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$
4. $\int \frac{du}{1+u^2} = \tan^{-1} u + C$
5. $\int \sec(u)\,du = \ln|\sec(u)+\tan(u)| + C$
6. $\int u\,dv = uv - \int v\,du$

Algebra formulas:

1. $\ln(xy) = \ln(x) + \ln(y)$
2. $a^{x+y} = a^x\,a^y$
3. $a^b = e^{b\ln a}$

Problem 1 (16 points)
Evaluate the integrals:

(a) $\int_1^2 x \ln(x)\, dx$

(b) $\int \cos^3(x)\, dx$

Problem 2 (10 points)

Evaluate the integral $\int \frac{5x-3}{(x+1)(x-3)}\,dx$.

Problem 3 (10 points)

A parabola centered at $(0,0)$ and opening upwards goes through the point $(-4,1)$.
Find an equation for this curve. What are the coordinates of its focus? (Write out the equation and the coordinates explicitly!)

Problem 4 (11 points)

A function $f(x)$ obtained from real-world measurements takes on these values:

x	$f(x)$
1	3
2	2
3	3
4	1

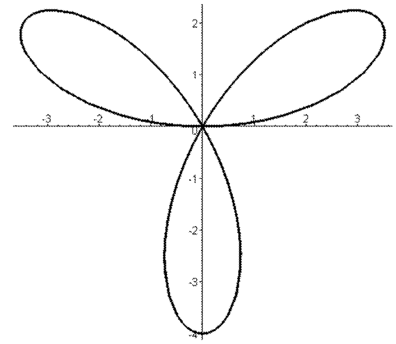
Estimate $\int_1^4 f(x)\,dx$ using one of our numerical integration techniques, Simpson's Rule or the Trapezoidal Rule: Be sure to specify which you are using!

Problem 5 (11 points)

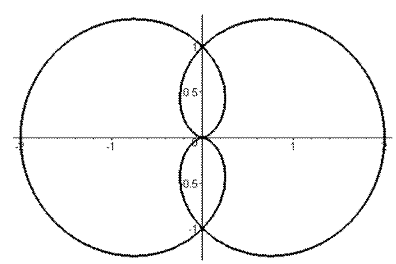
Find parametric equations $x = f(t)$ and $y = g(t)$ describing motion along the hyperbola $-\frac{x^2}{16} + \frac{y^2}{4} = 1$, such that the point (x, y) is at $(0, -2)$ when $t = 0$ and it moves to the right as t increases.

Problem 6 (14 points)

(a) For the curve $r = 4\sin\ 3\theta$, find the slope where $\theta = \frac{\pi}{4}$. The plot below and to the right shows roughly what this curve looks like: You must calculate the slope using derivatives to get credit.

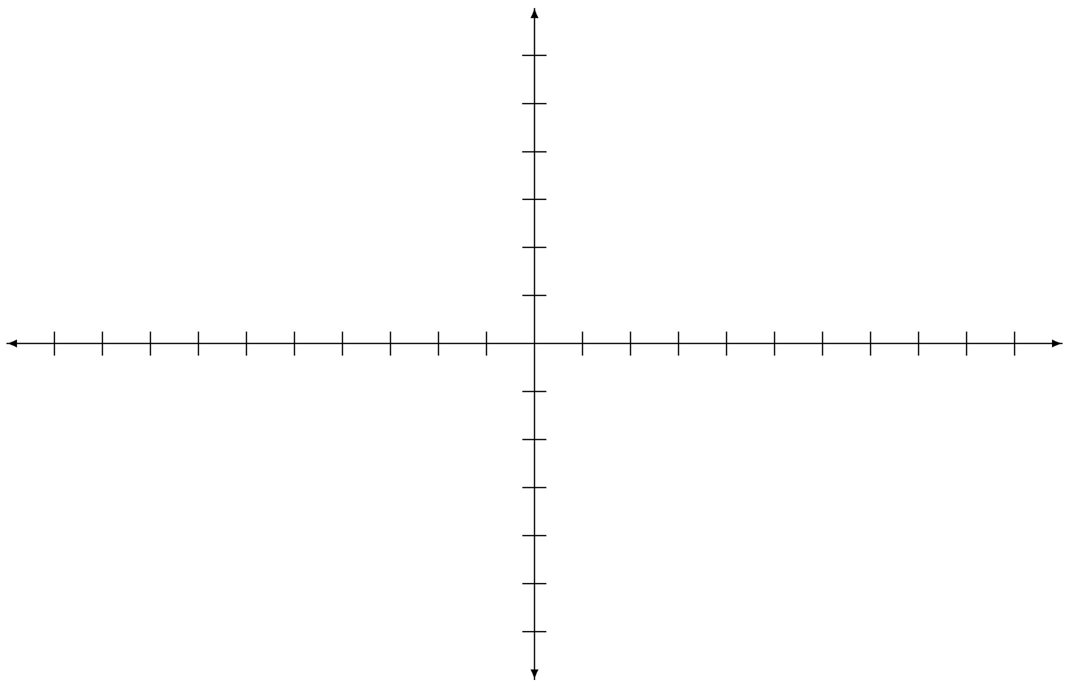


(b) Find the points where $r = 1 + \cos\ \theta$ and $r = 1 - \cos\ \theta$ intersect. You can use the plot at the right below as a check of your work but you must show how you calculate the specific coordinates of each intersection point: Just reading the intersection points from the plot will not receive any credit.



Problem 7 (12 points)

(a) Sketch the curve $\frac{x^2}{9} + \frac{y^2}{25} = 1$. Be sure to show where it crosses the x-axis and/or the y-axis (write out the coordinates!), and where its foci are (write out the coordinates!). Your sketch will not be graded for drawing ability but should resemble the correct curve.



(b) The equation $4x^2 + 2\sqrt{3}\,xy + 2y^2 + 10\sqrt{3}\,x + 10y = 5$ describes a conic section. (An ellipse, parabola, or hyperbola, not a degenerate case such as a line or point.)

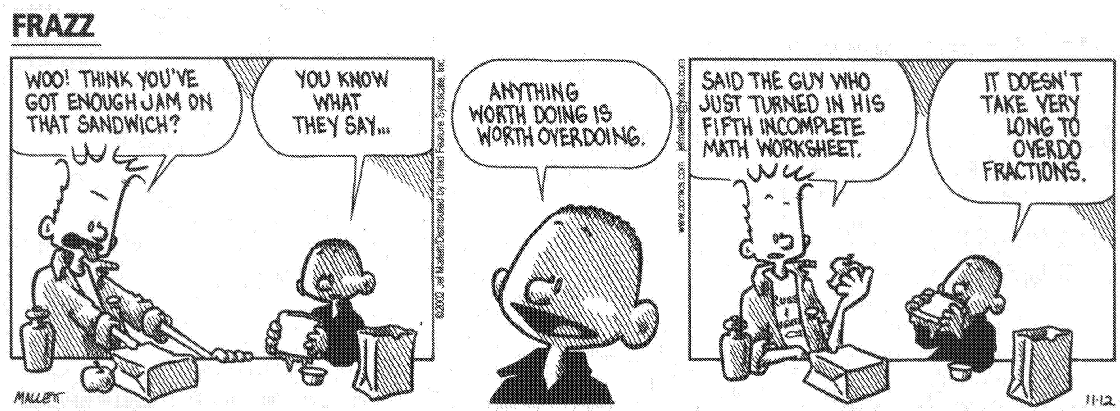
(i) Find an angle θ such that rotation of the coordinate system by θ would eliminate the xy term. (You do not need to carry out the rotation!)

(ii) Which kind of curve (ellipse, parabola, or hyperbola) is this conic section?

Problem 8 (16 points)

(a) Evaluate the integral $\int \frac{x^3\,dx}{\sqrt{16-x^2}}$.

(b) One of the integrals $\int_1^\infty \frac{dx}{\sqrt{x}}$ and $\int_1^\infty \frac{dx}{x^3}$ converges, and the other does not. Evaluate the one that converges.



Scratch Paper

(not a command!)