Mathematics 222, Lecture 2 (Wilson) **Your Name:** ____________________
Circle your TA's name:

Ethan Berkove **Youngwoo Choi** **Joseph Duffy**

Daode Huang **Jonathan Kochavi** **Chang Ock Lee**

Exam IIa 11/11/93

PLEASE NOTE: This is not the exam which was written to be given this evening. There is evidence that someone, somehow, stole a copy of that exam and has been distributing it, possibly for money. This exam is intended to be of equal difficulty to the original exam. It also covers the material you were told to study.

The original exam was created over several days and given careful proofreading as well as review of its content and difficulty. For this one I had only a few minutes. I have done the best I can to give you an opportunity to show what you know: If there are "typos" or other errors I am sorry. Bob Wilson

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Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem. Don't be afraid to use *words* in explaining your answers, but also don't try to use them to cover up if you don't know what you are talking about...
You may refer to notes you have brought in on one 4" by 6" index card, as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	12	
2	12	
3	13	
4	13	
5	13	
6	12	
7	13	
8	12	
TOTAL	100	

Problem 1 (12 points)

Tell whether the following series converge absolutely, conditionally, or not at all, explaining in detail why that is so:

(i)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{\frac{3}{2}}}$$

(ii)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{\frac{2}{3}}}$$

(iii)

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{2^n}$$

(iv)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\sqrt{n}}$$

Problem 2 (12 points)

(a) (8 points)

(i) Find the Maclaurin series for $f(x) = e^{2x} - \cos(x)$. Show what the general term looks like: Do not just write down a few terms and some dots.

(ii) What is the interval of convergence for the series you got in (i)? For what points does it converge absolutely? For what points does it converge conditionally?

(b) (4 points)

Find the Taylor polynomial of degree 4 for $f(x) = x^4$ centered at the point $a = 1$. Simplify this polynomial as much as possible: Write it in powers of x, not of $x - 1$.

(There is a very short way to do this problem, which is quite acceptable if provided with justification. You can also work it out in a straightforward manner.)

Problem 3 (13 points)

The first several terms of the Maclaurin series for e^{-x} are $1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24}$. If this expression is used to calculate an approximate value for $e^{-.2}$, how accurate is the result? Give a bound for the error, with justification. (The error bound and justification must be based on Taylor's theorem with remainder or a similar rigorous argument: Merely comparing the result of evaluating the expression above with the result your calculator gives for $e^{-.2}$ will not receive *any* credit.)

Problem 4 (13 points)

We need to calculate the value of $\cos(x)$ at $x = 0.2$, and we decide to use the Maclaurin series for $\cos(x)$ to do the job. We need the answer accurate to within plus or minus 0.0001. How many terms of the series do we need to use? Justify your answer.

(The calculations determining the number of terms must be based on Taylor's theorem with remainder or a similar rigorous argument: Merely evaluating different polynomials and comparing the results with the result your calculator gives for $\cos(0.2)$ will not receive *any* credit.)

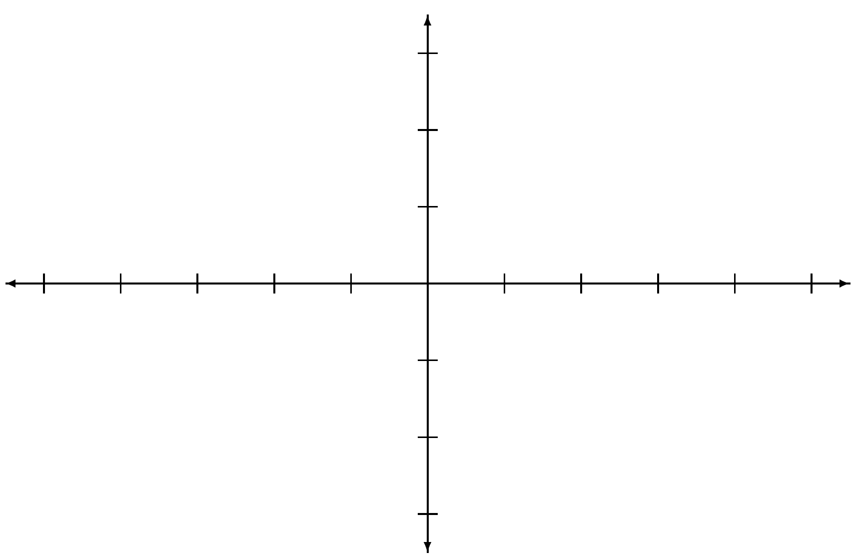
Problem 5 (13 points)
Sketch the graph for the equation

$$9x^2 + 36x - 16y^2 + 32y - 124 = 0$$

Be sure to show or tell, with numeric values where appropriate:

- What kind of curve is this?
- Where the curve crosses the axes, if there are such points.
- The axes of the figure, if any.
- The vertices of the figure, if any.
- The focus or foci of the figure, if any.
- The eccentricity of the figure, if that applies.

You may change the axes below, so that one mark is possibly not one unit: Be sure to label the axes.



Problem 6 (12 points)

(a) (6 points) For each of the equations below: By what angle would you rotate the axes to convert the equation to one "without an xy term"? (If you cannot express the angle in radians, you may wish to describe it using an inverse trig function.) Tell which conic section gives the graph of the equation. Note that you are not required to tell any other facts about the curve or to graph it.

(i)

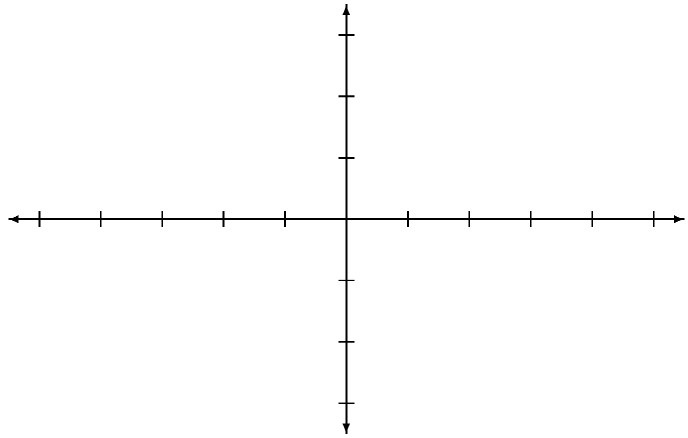
$$2x^2 + 4xy - y^2 - 2x + 3y - 6 = 0$$

(ii)

$$3x^2 + 6xy + 3y^2 - 4x + 5y - 12 = 0$$

(b) (6 points) Rotate the coordinate axes to convert the following equation to one "without an xy term". Then sketch the graph of the conic section it describes.

$$11x^2 + 10\sqrt{3}xy + y^2 = 4$$



Problem 7 (13 points)

(a) (8 points)

A particle moves along the curve $9x^2 + 16y^2 = 144$.

(i) Find a parametrization of the curve which corresponds to the particle making one trip around the curve in a clockwise direction starting at $(4, 0)$.

(ii) Find a parametrization of the curve which corresponds to the particle making one trip around the curve in a counterclockwise direction starting at $(0, 3)$.

(b) (5 points)

Find an equation for the tangent line to the curve parameterized by $x = 2\sin(t)$, $y = \cos(2t)$, at the point where $t = \frac{\pi}{4}$.

Problem 8 (12 points)

Find the area of the surface generated by revolving the curve given by $x = \frac{t^2}{2}$, $y = 2t$ for $0 \leq t \leq \sqrt{5}$ around the x-axis.

SCRATCH PAPER