Your Name: ______________________________

Circle your TA's name:

Clark Good	**Aaron Greenblatt**	**Sharib Haroon**
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Mathematics 222, Fall 2001 **Lecture 1 (Wilson)**

Third Midterm Exam December 4, 2001

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using π, $\sqrt{3}$, $\cos^{-1}(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is scratch paper at the end of the exam. If you need more scratch paper, please ask for it.

You may refer to one or two pages of notes you have brought with you, as announced in class.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" are not sufficient substantiation...)

Problem	Points	Score
1	13	
2	13	
3	12	
4	12	
5	12	
6	12	
7	14	
8	12	
TOTAL	100	

Some formulas, identities, and numeric values you might find useful:

Values of trig functions:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	—

Trig facts:

1. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
2. $\sec\theta = \frac{1}{\cos\theta}$
3. $\sin^2\theta + \cos^2\theta = 1$
4. $\sec^2\theta = \tan^2\theta + 1$
5. $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
6. $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
7. $\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$
8. $\sin^2 x = \frac{1}{2}(1-\cos 2x)$
9. $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

Derivative formulas:

1. $\frac{d}{dx}\tan x = \sec^2 x$
2. $\frac{d}{dx}\sec x = \sec x\,\tan x$
3. $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
4. $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$
5. $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$
6. $\frac{d}{dx}\ln x = \frac{1}{x}$
7. $\frac{d}{dx}e^x = e^x$

Integral formulas: (We assume you know, and you are certainly allowed to use, basic formulas for integrals of functions such as x^n, e^x, $\sin x$, $\cos x$, etc., and how to use substitution to extend these.)

1. $\int \frac{1}{u}\,du = \ln|u| + C$
2. $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$
3. $\int \frac{du}{1+u^2} = \tan^{-1} u + C$
4. $\int \sec(u)\,du = \ln|\sec(u) + \tan(u)| + C$
5. $\int u\,dv = uv - \int v\,du$

Algebra formulas:

1. $\ln(xy) = \ln(x) + \ln(y)$
2. $a^{x+y} = a^x\,a^y$
3. $a^b = e^{b\ln a}$

Problem 1 (13 points)
For the series

$$3 - 1 + \frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \cdots$$

(a) Find a formula for the n^{th} partial sum of this series.

(b) This series does converge. To what does it converge?

Problem 2 (13 points)
For the function $f(x) = \cos 2x$, write the Taylor series for $f(x)$ at $a = \frac{\pi}{4}$: You do not need to show what the n^{th} degree term looks like in general, but you should include all terms with degree ≤ 6.

<u>Problem 3</u> (12 points)
For each of the following series, tell whether the series converges and give a reason showing that it converges or diverges as you have claimed.

(a) $$\sum_{n=2}^{\infty} \frac{n}{n^2-1}$$

(b) $$\sum_{n=1}^{\infty} \frac{2^n+3}{5^n}$$

(c) $$\sum_{n=1}^{\infty} \cos\left(\frac{n\,\pi}{2}\right)$$

(d) $$\sum_{n=1}^{\infty} (-1)^n \frac{1}{1-\ln n}$$

Problem 4 (12 points)
The series

$$f(x) = \sum_{n=0}^{\infty} 2(-2x)^n \qquad \Big(= 2 - 4x + 8x^2 - 16x^3 + 32x^4 - \cdots\Big)$$

can be used to define a function $f(x)$, wherever the series converges.

(a) What are the interval of convergence and the radius of convergence for this power series?

(b) For the values of x where this series converges, give a formula for $f(x)$ which does not involve a series. (I.e., a formula involving pieces which are rational, trigonometric, exponential or logarithmic functions.)

Problem 5 (12 points)

For each of the following series: (i) Tell whether it converges; (ii) Tell whether it converges absolutely; and (iii) Tell whether it converges conditionally. Be sure to give reasons justifying your answers, and indicate clearly all three answers for each series.

(a) $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\, 2^n}{n!}$$

(b) $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\, n!}{e^n}$$

(c) $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n - \sin n}$$

Problem 6 (12 points)
Let $\vec{u} = -2\vec{\imath} + 6\vec{\jmath}$ and $\vec{v} = 4\vec{\imath} - 5\vec{\jmath}$.

(a) Find $\vec{u} + 4\vec{v}$.

(b) What is the length of the vector $3\vec{u}$? (Your answer should be a number. It may have square roots in it.)

(c) If A is the point with coordinates $(2, 3)$, for what point B is $\vec{u} = \overrightarrow{AB}$?

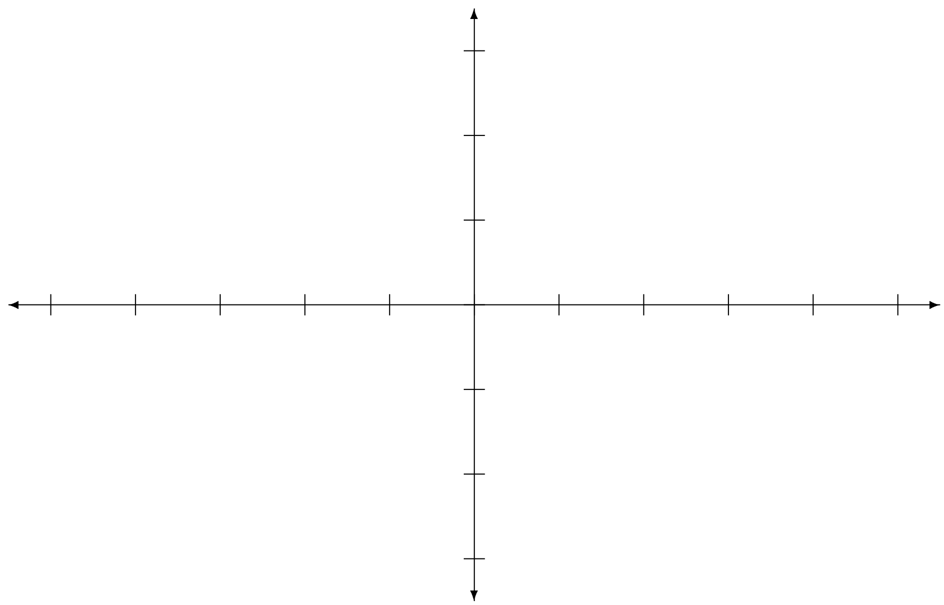
(d) Find a vector of unit length in the same direction as $\vec{v}$.

Problem 7 (14 points)

You need to find a polynomial approximation to $\sin(2x)$ which will work for all the values of x between ± 0.1 with an error no larger than 0.0001. Find such a polynomial using initial terms from the Maclaurin series for $\sin(2x)$: Tell explicitly what polynomial you use, showing what the terms are and of course what the degree is, and show that it meets the accuracy requirement.

Problem 8 (12 points)

(a) A point has coordinates obeying the parametric equations $x = t^2$ and $y = \frac{t}{2} + 1$, for $-2 \leq t \leq 6$. Find a Cartesian equation (either $y = f(x)$ or $x = f(y)$) describing the curve this point moves along, and sketch the path of the point for the given values of t.



(b) Find parametric equations which correspond to traversing the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ exactly once, starting at the point $(-3, -4)$ and going in a clockwise direction.

SCRATCH PAPER