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Mathematics 222, Fall 2001

Lecture 1 (Wilson)

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Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using π , $\sqrt{3}$, $\cos^{-1}(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

You may refer to one or two pages of notes you have brought with you, as announced in class.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" are not sufficient substantiation...)

Problem	Points	Score
1	10	
2	10	
3	12	
4	12	
5	12	
6	16	
7	16	
8	12	
TOTAL	100	

Some formulas, identities, and numeric values you might find useful:

Values of trig functions:

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	—

Trig facts:

1. $\tan \theta = \frac{\sin \theta}{\cos \theta}$
2. $\sec \theta = \frac{1}{\cos \theta}$
3. $\sin^2 \theta + \cos^2 \theta = 1$
4. $\sec^2 \theta = \tan^2 \theta + 1$
5. $\sin(x + y) = \sin(x) \cos(y) + \cos(x) \sin(y)$
6. $\cos(x + y) = \cos(x) \cos(y) - \sin(x) \sin(y)$
7. $\tan(x + y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \tan(y)}$
8. $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
9. $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

Derivative formulas:

1. $\frac{d}{dx} \tan x = \sec^2 x$
2. $\frac{d}{dx} \sec x = \sec x \tan x$
3. $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
4. $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$
5. $\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$
6. $\frac{d}{dx} \ln x = \frac{1}{x}$
7. $\frac{d}{dx} e^x = e^x$

Integral formulas: (We assume you know, and you are certainly allowed to use, basic formulas for integrals of functions such as x^n , e^x , $\sin x$, $\cos x$, etc., and how to use substitution to extend these.)

1. $\int \frac{1}{u} du = \ln |u| + C$
2. $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$
3. $\int \frac{du}{1+u^2} = \tan^{-1} u + C$
4. $\int \sec(u) du = \ln |\sec(u) + \tan(u)| + C$
5. $\int u dv = uv - \int v du$

Algebra formulas:

1. $\ln(xy) = \ln(x) + \ln(y)$
2. $a^{x+y} = a^x a^y$
3. $a^b = e^{b \ln a}$

Problem 1 (10 points)

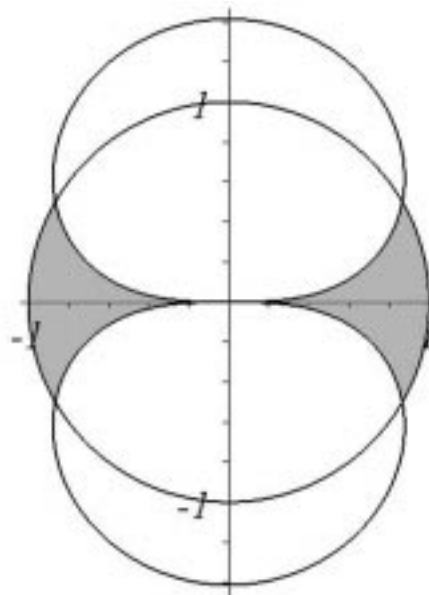
Solve the differential equation $e^{-x} \frac{dy}{dx} = 1 + y^2$. You should find the general solution, in the form $y = f(x)$ for some f .

Problem 2 (10 points)

Find the center (in x - y coordinates) and radius of the circle $r = 6 \cos(\theta) - 4 \sin(\theta)$.

Problem 3 (12 points)

Find the area of the region which is inside the circle $r = 1$ but outside the curve $r^2 = 2 \sin \theta$.



Problem 4 (12 points)

(a) Find all solutions of $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 20y = 0$.

(b) Describe what happens to the solutions found in (a) as $t \rightarrow \infty$.

Problem 5 (12 points)

Solve the initial value problem: $x\frac{dy}{dx} + y = \sin x$, with $y(\frac{\pi}{2}) = 3$. (You may assume $x > 0$.)

Problem 6 (16 points)

This problem essentially asks you to solve the initial value problem $y'' - 3y' + 2y = 5e^x$ with $y(0) = 0$ and $y'(0) = -7$. You are to find the solution in several steps as outlined below, and we give you answers at several stages. Thus if you make a mistake in one part you should still be able to complete the other parts correctly. But: You get no points for finding a solution which we give you, so what counts is showing how to get the solution.

(a) Solve $y'' - 3y' + 2y = 0$.

(The general solution to this homogeneous equation is $y_h = C_1e^x + C_2e^{2x}$: You should show how to find it, not just show that this is a solution by testing it in the equation.)

(b) Find a particular solution to $y'' - 3y' + 2y = 5e^x$.

(One solution is $y_p = -5xe^x$. As in part (a) you must show how to find y_p , not just check that this is a solution. You may find a different solution but it must be a solution.)

(c) Find all solutions to $y'' - 3y' + 2y = 5e^x$.

(d) Find the solution to $y'' - 3y' + 2y = 5e^x$ with $y(0) = 0$ and $y'(0) = -7$.

Problem 7 (16 points)

(a) Write a formula describing the n^{th} term a_n of a sequence if the first 5 terms are

$$1, \quad -\frac{1}{3}, \quad \frac{1}{5}, \quad -\frac{1}{7}, \quad \text{and} \quad \frac{1}{9}.$$

(b) The sequence $a_n = \frac{1}{n} \cos(n)$ converges. What is its limit?

(c) Does the sequence $a_n = \sin(n\pi)$ converge? Give a reason. If it does converge, what is the limit?

(d) Does the sequence $a_n = \sin(n\pi + \frac{\pi}{2})$ converge? Give a reason. If it does converge, what is the limit?

Problem 8 (12 points)

- (a) Suppose a polar curve is given by $r = e^\theta$. Show that the angle ψ between the tangent line and the radius vector, at any point (r, θ) on the curve, is $\frac{\pi}{4}$.

- (b) Suppose a polar curve, not necessarily the same one as in (a), is given by $r = f(\theta)$. Further suppose that the angle ψ for this curve is some constant α , not necessarily $\frac{\pi}{4}$, at every point (r, θ) on the curve. Show that $f(\theta)$ must be $Ce^{k\theta}$ for some constants k and C . (I.e., find in terms of α , a number k such that $f(\theta)$ must be $Ce^{k\theta}$ for some C .)