Your Name: _____

Circle your TA's name:

Clark Good	**Aaron Greenblatt**	**Sharib Haroon**
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Mathematics 222, Fall 2001 **Lecture 1 (Wilson)**

First Midterm Exam October 2, 2001

Write your answers to the seven problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using π, $\sqrt{3}$, $\cos^{-1}(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is scratch paper at the end of the exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on a sheet of paper, as announced in class.

On the other side of this sheet there is a collection of facts and formulas. In a problem where you are asked to evaluate an integral, you may directly use these formulas. If the function you are asked to integrate does not directly fit these formulas, you must show all work needed to do the integral using these formulas together with substitution and the techniques studied in chapter seven of the text. Do not simply quote some other formula and "plug in" the function.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" are not sufficient substantiation...)

Problem	Points	Score
1	16	
2	15	
3	16	
4	14	
5	14	
6	11	
7	14	
TOTAL	100	

Some formulas, identities, and numeric values you might find useful:

Values of trig functions:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	—

Trig facts:

1. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
2. $\sec\theta = \frac{1}{\cos\theta}$
3. $\sin^2\theta + \cos^2\theta = 1$
4. $\sec^2\theta = \tan^2\theta + 1$
5. $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
6. $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
7. $\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$
8. $\sin^2 x = \frac{1}{2}(1-\cos 2x)$
9. $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

Derivative formulas:

1. $\frac{d}{dx}\tan x = \sec^2 x$
2. $\frac{d}{dx}\sec x = \sec x\,\tan x$
3. $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
4. $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$
5. $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$
6. $\frac{d}{dx}\ln x = \frac{1}{x}$
7. $\frac{d}{dx}e^x = e^x$

Integral formulas:

1. $\int u^n\,du = \frac{1}{n+1}u^{n+1} + C$, if $n \neq -1$
2. $\int \frac{1}{u}\,du = \ln|u| + C$
3. $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$
4. $\int \frac{du}{1+u^2} = \tan^{-1} u + C$
5. $\int \sec(u)\,du = \ln|\sec(u) + \tan(u)| + C$
6. $\int u\,dv = uv - \int v\,du$

Algebra formulas:

1. $\ln(xy) = \ln(x) + \ln(y)$
2. $a^{x+y} = a^x\,a^y$
3. $a^b = e^{b\ln a}$

Problem 1 (16 points)

Evaluate the integrals: Note the instructions on the first page of the exam regarding use of formulas!

(a) $\int x^2 e^{-x}\, dx$

(b) $\int_0^2 \frac{x^2}{\sqrt{8-x^2}}\, dx$

Problem 2 (15 points)
For the conic section

$$9x^2 + 25y^2 = 225$$

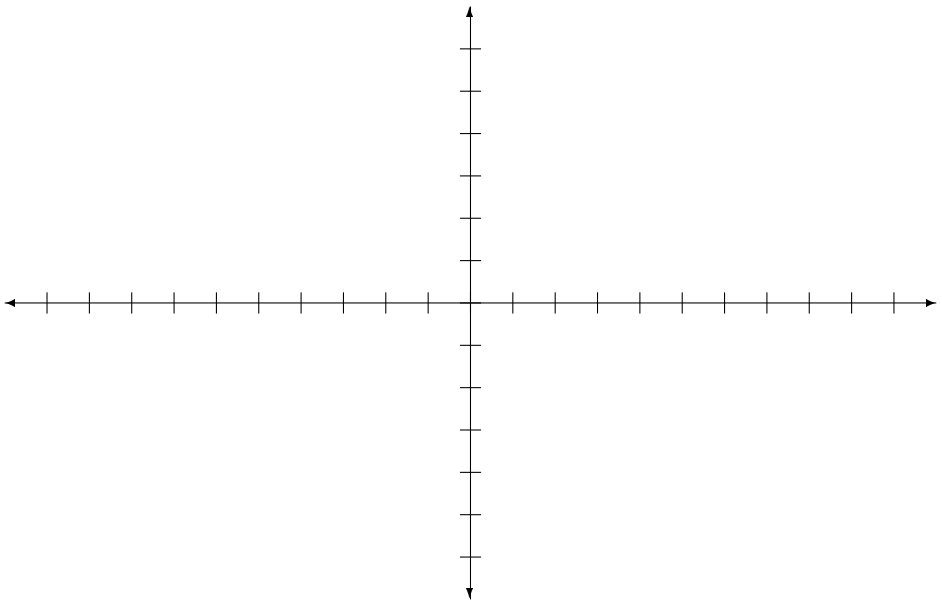
(a) Find the coordinates of the foci.

(b) Find the coordinates of the points where this curve extends furthest in the positive and negative x directions.

(c) Find the coordinates of the points where this curve extends furthest in the positive and negative y directions.

(d) What is the eccentricity?

(e) Sketch the curve on the axes below. Be sure to label both axes to show units.



Problem 3 (16 points)

Evaluate the integrals: Note the instructions on the first page of the exam regarding use of formulas!

(a) $\int \sin^3 x \, \cos^{3/2} x \, dx$

(b) $\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}} \, dx$

Problem 4 (14 points)

(a) Find an equation for the hyperbola which has foci $(0, \pm 13)$ and asymptotes $y = \pm \frac{12}{5}\, x$.

(b) What is the eccentricity of this hyperbola?

Problem 5 (14 points)
Evaluate the integral: Note the instructions on the first page of the exam regarding use of formulas!

$$\int \frac{7x^2 - 6x + 16}{(x-2)(x^2+4)}\, dx$$

Problem 6 (11 points)
Consider the equation:

$$3x^2 - 2xy + 5y^2 - 7x + y - 2 = 0$$

(a) Find an angle α such that rotating the coordinate system by α transforms the equation so that the xy term disappears. (You do not need to carry out the coordinate rotation.)

(b) What kind of conic section is the graph of this equation?

Problem 7 (14 points)
Find the area between the graph of $f(x) = \ln x$ and the x-axis, for $\frac{1}{2} \leq x \leq 2$.

SCRATCH PAPER