

Mathematics 221, Lecture 6 (Wilson)

Your Name: _____

Circle your TA's name:

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Final Exam 12/22/99

There are problems on the back of this sheet!

Write your answers to the twelve problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using π , $\sqrt{3}$, and similar numbers) rather than using decimal approximations.

There is scratch paper on the back of the exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on three index cards, as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	17	
2	16	
3	16	
4	17	
5	17	
6	17	
7	17	
8	16	
9	16	
10	17	
11	17	
12	17	
TOTAL	200	

Problem 1 (17 points)

The points satisfying $\frac{x^2}{36} + \frac{y^2}{9} = 1$ form an ellipse. Find an equation for the tangent line to this ellipse at the point $(2, 2\sqrt{2})$.

Problem 2 (16 points)

An object moves along the y -axis so that its velocity at time t is $v(t) = 12t^2 - 5t + 2$. When $t = 2$, the object is at $y = 5$. All distance measurements are in inches, and time is measured in seconds.

(a) What is the object's acceleration $a(t)$ in general? What is the acceleration when $t = 2$? (Give units!)

(b) What is the position $s(t)$ of the object in general? Where is it along the y -axis when $t = 4$? (Give units!)

Problem 3 (16 points)

Evaluate the definite or indefinite integrals:

(a)
$$\int_0^2 \frac{2x^2}{x^3 + 1} dx$$

(b)
$$\int \frac{e^{3x}}{\sqrt{1 - e^{6x}}} dx$$

(c)
$$\int_{-5}^0 (|x + 4| - 1) dx$$

(d)
$$\int (5s^3 + 4s^2 - s + 6) ds$$

Problem 4 (17 points)

(a) Find $F'(x)$ for $F(x) = \int_{\pi}^x (t^2 - 2) dt$.

(b) Find $F'(x)$ for $F(x) = \int_{\sin(x)}^5 e^{t^2} dt$.

Problem 5 (17 points)

Find the area of the region bounded by $y = |x|$ and $y = x^4$. (Warning: There are two parts to this region!)

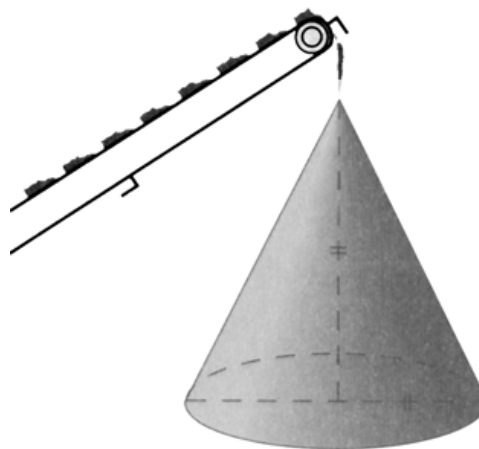
Problem 6 (17 points)

The population in a colony of bacteria grows at a rate proportional to the population itself. The population is observed to double in 7 hours. At noon there are 100,000 bacteria. What is the population at 4:30 PM?

Problem 7 (17 points)

A conveyor belt is carrying gravel up and dumping it on the top of a pile. The gravel is such that the pile always has the shape of a circular cone whose base diameter is the same as its height. The conveyor belt adds 30 cubic feet of gravel to the pile each minute. How fast is the height of the pile growing when the pile is 10 feet high?

A formula for the volume of a cone is $\frac{1}{3}Ah$ where A is the area of the base of the cone and h is the height of the cone.



Problem 8 (16 points)

The region between the curve $y = \sin(x)$ and the x -axis for $0 \leq x \leq \frac{\pi}{2}$ is rotated about an axis to produce a solid object.

- (a) Suppose the region is rotated about the x -axis. Set up but **DO NOT EVALUATE** an integral to find the volume of the resulting object.

- (b) Suppose the region is rotated about the line $x = -3$. Set up but **DO NOT EVALUATE** an integral to find the volume of the resulting object.

Problem 9 (16 points)

(a) Find $f'(x)$ for $f(x) = \frac{\ln(x)}{1+x^2}$.

(b) Find $\frac{dy}{dx}\big|_{x=\frac{\pi}{4}}$ for $y = e^{\cos(2x)}$.

(c) Find $f'(x)$ for $f(x) = \sin^{-1}(\tan(x))$.

(d) Find $f''(\frac{\pi}{2})$ for $f(x) = x^2 + \sin(x)$.

Problem 10 (17 points)

The table gives values of a function $g(x)$:

x	0	5	10	15	20	25	30	35	40
$g(x)$	7.1	12	3.4	0	-2	-4.1	-5	-12	-18.3

- (a) Estimate the value of $\int_0^{40} g(x) dx$ using a Riemann sum with four equal-length subintervals and using the midpoint of each subinterval as x_i^* .

- (b) Describe conditions under which the sum you got in (a), divided by 40, would give exactly the average value of $g(x)$ on the interval $[0, 40]$.

Problem 11 (17 points)

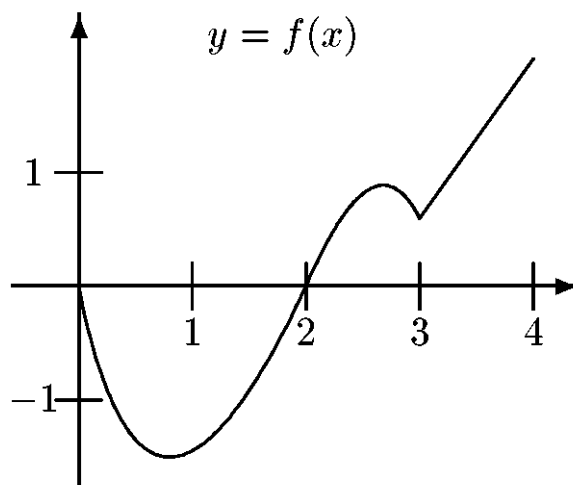
A cylindrical can is to be made from 54π square inches of sheet metal. (That metal must suffice for the cylindrical portion of the can and for both circular ends.)

- (a) What dimensions (radius and height) should the can have in order to maximize the volume in the can?

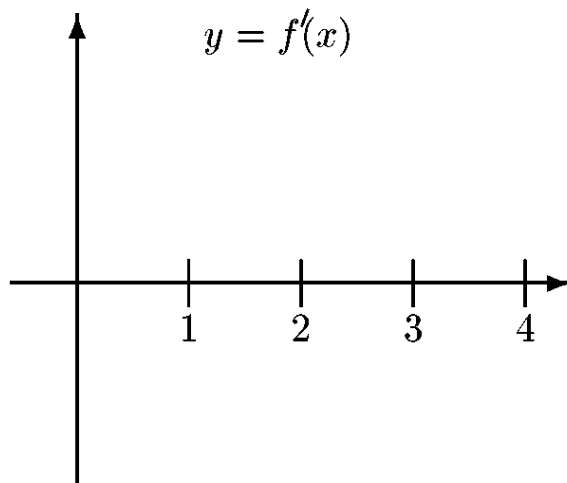
- (b) What is that maximum volume?

Problem 12 (17 points)

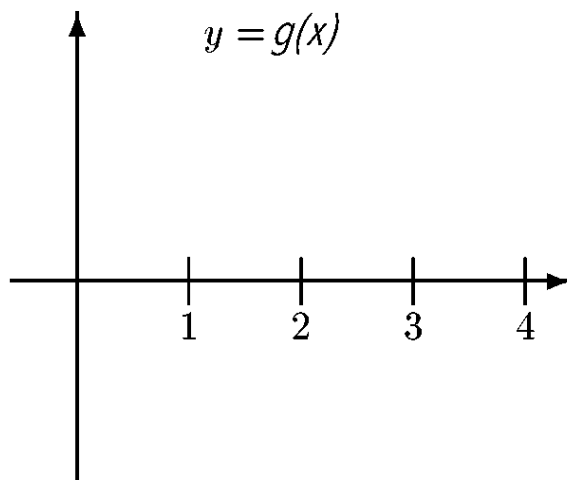
The graph to the right shows $f(x)$:



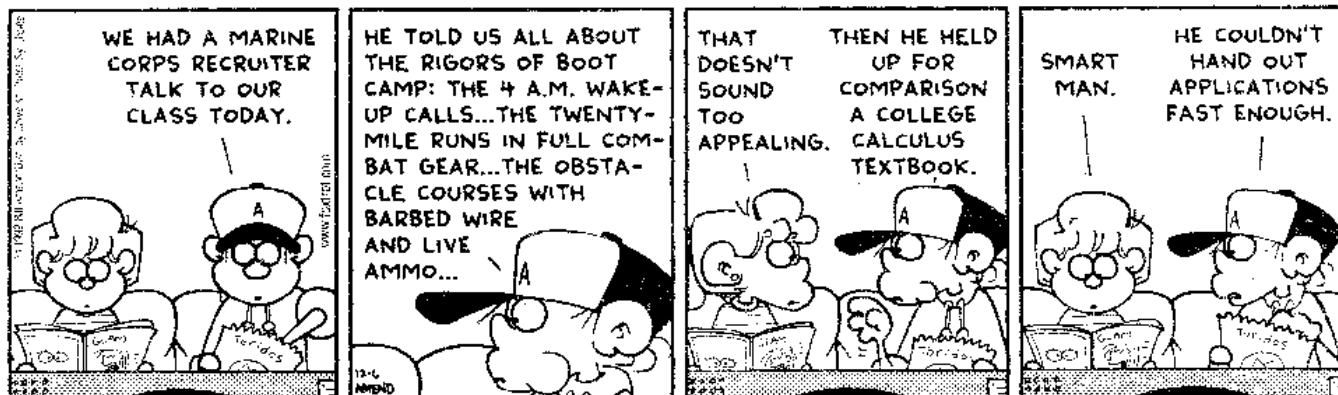
Sketch a graph of $f'(x)$:



Let $g(x) = \int_0^x f(t) dt$. Sketch a graph of $g(x)$:
Label the maxima and minima and the points of inflection of $g(x)$.



Foxtrot



SCRATCH PAPER