Mathematics 221, Lecture 6 (Wilson)

Your Name: ______________________________

Circle your TA's name:

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Susan Hollingsworth	**Sarah Kerbeshian**	**Peter Spaeth**

Exam II 11/18/99

Write your answers to the nine problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using π, $\sqrt{3}$, and similar numbers) rather than using decimal approximations. There is scratch paper on the back of this sheet. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on two index cards, as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	15	
2	10	
3	10	
4	10	
5	10	
6	14	
7	10	
8	8	
9	13	
TOTAL	100	

SCRATCH PAPER

Problem 1 (15 points)

(a) For $y = \sin(\ln(x^2 - 2))$, find $\frac{dy}{dx}$.

(b) For $f(x) = e^{3x}\cos(x)$, find $f'(0)$.

(c) For $f(x) = \tan^{-1}(2x)$, find $\frac{d^2 f}{dx^2}$.

(d) Find $\frac{dy}{dx}$ if $y = x^{2x+1}$.

(e) Find an equation for the tangent line to the graph of $e^{x-y} = 2y + 1$ at the point $(0, 0)$.

Problem 2 (10 points)

A bacterial population numbers one million at the start of an experiment. The population grows at a rate proportional to its size at any given time. If the population after 2 hours is three million:

(a) Find a function $P(t)$ which gives the population at any time t (measured in hours after the start of the experiment.)

(b) What is the population 3 hours after the experiment starts?

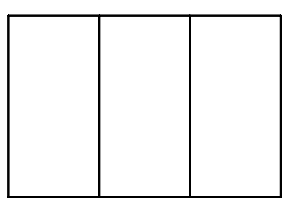
(c) How fast is the population growing when $t = 3$? What are the units on this rate of growth?

Problem 3 (10 points)

We want to fence in a field which is rectangular, and which is divided into three parts by additional fencing parallel to one of the sides of the field. There is a fixed amount, 2000 feet, of fence available to be used for all parts of the fencing. What should be the dimensions of the field be in order to maximize the area inside the fence?

The drawing below shows the fences, with the interior fences dividing the overall field into three equal parts. The equality actually does not matter for this problem.

(Be sure to show how you set up this problem and work it out. Just having the final numeric answer will not get any credit. You should give some reason for believing your answer is a maximum rather than a minimum.)



Problem 4 (10 points)
Evaluate the limits: (Be sure to show how you arrive at your answers. You should do more than just look at a sequence of numbers from a calculator.)

(a) $$\lim_{x \to 0} \frac{\tan(x)}{x - 2\sin(x)}$$

(b) $$\lim_{x \to \infty} (\sqrt{x^2 + 4} - \sqrt{x^2 - 3x})$$

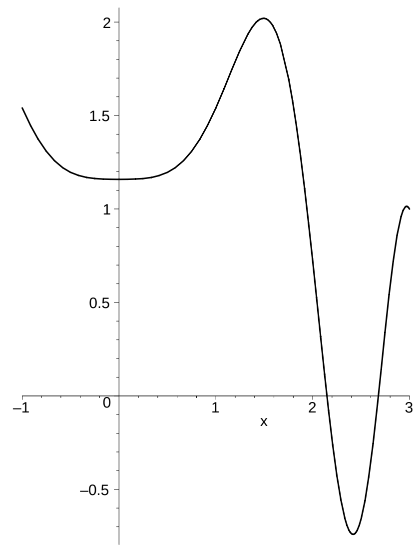
Problem 5 (10 points)
Point B is 100 miles straight north of point A. A car leaves point A at noon and drives straight north at 50 miles per hour. A second car leaves point B one hour later and drives straight east at 30 miles per hour.

(a) When are the two cars closest to each other?

(b) How far apart are the cars when they are closest to each other?

Problem 6 (14 points)

The figure at the right shows the graph of $f'(x)$. (Note that this is not the graph of $f(x)$.) Use this graph to answer the questions below about the function f. Give numeric answers to the nearest tenth of a unit. You do not need to give explanations for your answers on this problem.



(a) On what intervals (within $[-1, 3]$) is $f(x)$ increasing?

(b) On what intervals (within $[-1, 3]$) is $f(x)$ decreasing?

(c) At what what x values (within $[-1, 3]$) does $f(x)$ have a local maximum?

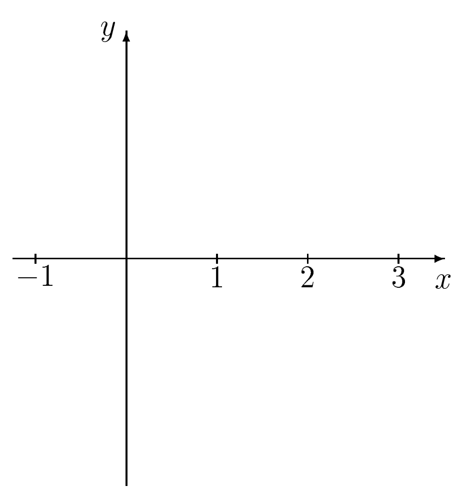
(d) At what what x values (within $[-1, 3]$) does $f(x)$ have a local minimum?

(e) At what what x values (within $[-1, 3]$) does $f(x)$ have a point of inflection?

(f) On what intervals (within $[-1, 3]$) is $f(x)$ concave up?

(g) On what intervals (within $[-1, 3]$) is $f(x)$ concave down?

(h) Sketch the graph of a function having all of the properties above and in addition satisfying $f(0) = 1$. Label the max/min points and points of inflection. Label the units on the y-axis.



Problem 7 (10 points)
Find all antiderivatives of:

(a) $x^3 + \frac{2}{\sqrt{x}}$

(b) $\frac{3}{1 + x^2}$

Problem 8 (8 points)
We are given that f is a function such that:

$$f''(x) = -2\sin(x) + 12x + 6$$

$$f(0) = 1 \qquad \text{and} \qquad f'(0) = 1.$$

What is $f(x)$?

Problem 9 (13 points)

x	$f(x)$
1.0	1.381773291
1.1	1.389212123
1.2	1.353816103
1.3	1.260402480
1.4	1.095178664
1.5	.8488103986
1.6	.5201559141
1.7	.1201022924
1.8	-.3254506885
1.9	-.7747553191
2.0	-1.172949332

The table at the right gives some values of the function $f(x) = \sin(x^2) + \cos(x)$.

(a) Use the intermediate value theorem to pick an interval in which the function f must have a root, i.e. an interval in which the theorem says there must be a number x with $f(x) = 0$. Tell how you use the theorem in picking the interval.

(b) Set up a calculation using Newton's method to locate the root more precisely. You should show what functions and derivatives are used in the calculation, and where they are evaluated, but you do not need to work out the numbers.

(c) Use a quadratic approximation to estimate the value of $f(1.11)$. Where values of the derivatives are used, show what needs to be calculated but you do not need to carry out the calculations.