

## Mathematics 221, Lecture 6 (Wilson)

Your Name: \_\_\_\_\_

Circle your TA's name:

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Exam I 10/7/99

Write your answers to the nine problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using  $\pi$ ,  $\sqrt{3}$ , and similar numbers) rather than using decimal approximations. There is scratch paper on the back of this sheet. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on one index card, as announced in class.

You may use a calculator, but remember:

**BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.**

Problem	Points	Score
1	10	
2	12	
3	12	
4	11	
5	11	
6	10	
7	11	
8	12	
9	11	
<b>TOTAL</b>	<b>100</b>	

# SCRATCH PAPER

Problem 1 (10 points)

Find an equation for the tangent line to the function  $f(x) = \cos(2x) - x$  at the point  $(0, 1)$ .

Problem 2 (12 points)

A particle moves along the  $y$ -axis so that its position at time  $t$  is given by  $y = 5t^2 - 3t + 2$ , where the position is measured in inches and the time in seconds. (Give units in your answers.)

(a) What is the average velocity of this particle between  $t = 1$  and  $t = 3$ ?

(b) What is the instantaneous velocity of this particle when  $t = 1.5$ ?

## Problem 3 (12 points)

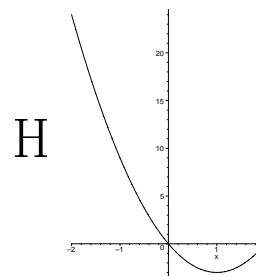
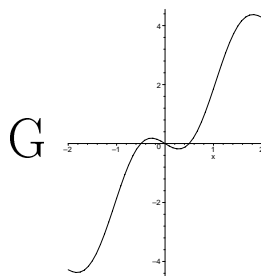
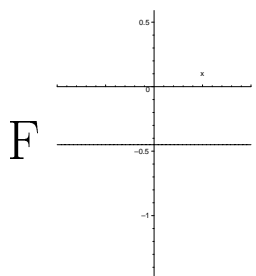
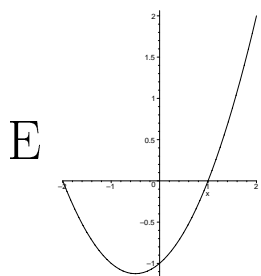
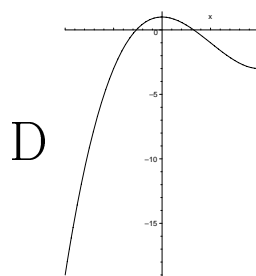
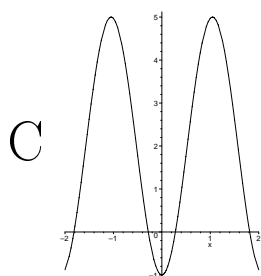
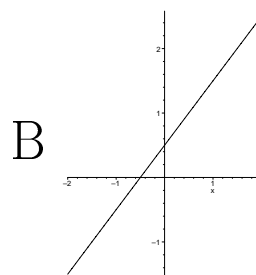
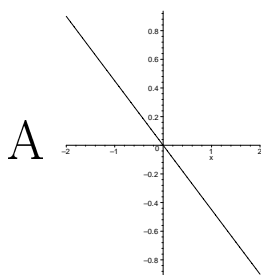
Of the eight functions graphed below, four are the derivatives of the other four. All are plotted with the same horizontal scale, but the vertical scales vary. Fill in the letters A-H in the blanks: each letter gets used just once.

Function \_\_\_\_\_ is the derivative of function \_\_\_\_\_.

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Problem 4 (11 points)

Use the definition of the derivative as a limit to find  $f'(1)$ , for  $f(x) = x^2 - 3x + 4$ .

Problem 5 (11 points)

(a) What is wrong with the statement  $\frac{x^2 - 3x + 2}{x - 2} = x - 1$  ?

(b) Modify the statement in part (a) to make it true, in a way that justifies the different statement

$$\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x - 2} = \lim_{x \rightarrow 2} (x - 1).$$

Problem 6 (10 points)

Let  $f(x) = \frac{x^2 - 4}{x + 2}$ .

Is  $f(x)$  continuous at  $x = -2$ ?

If so, justify your answer.

If not, tell what is wrong and change the function minimally so as to make it continuous.

Problem 7 (11 points)

(a) If  $\cos(y) = x^2 + 3$ , find  $\frac{dy}{dx}$ .

(b) Find the limit:  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 4x + 5}}{6x + 1}$

Problem 8 (12 points)

For each of the following, find  $\frac{dy}{dx}$ . (You do not need to use the limit-definition for these derivatives.)

(a)  $y = x \cos(x)$ .

(b)  $y = \frac{x^3}{1-3x}$ .

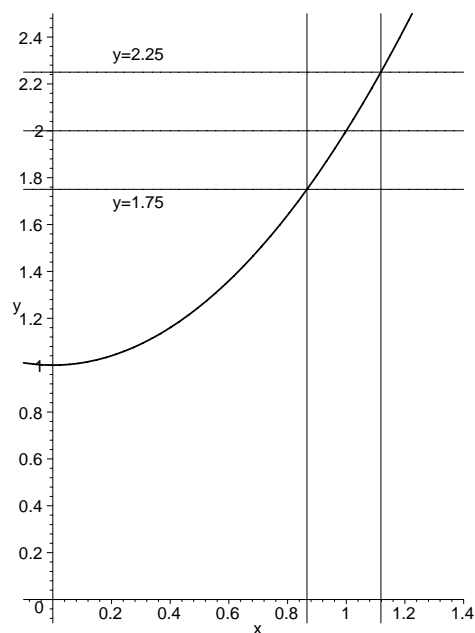
(c)  $y = x^2 + \sin(x^2)$ .

(d)  $y = \sqrt{3x^2 + 2}$ .

## Problem 9 (11 points)

Use the graph at the right, of the function  $f(x) = 1 + x^2$ , to find a number  $\delta$  such that:

$$|(1 + x^2) - 2| < \frac{1}{4} \text{ whenever } |x - 1| < \delta$$



Explain how you found  $\delta$ , and tell why your number fulfills the requirements above.