

Your Name: _____

Circle your TA's name:

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Mathematics 221, Summer 2009

(Wilson)

Final Exam August 6, 2009

Morning portion of the exam

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is a problem on the back of this sheet: Be sure not to skip over it by accident!

There is scratch paper at the end of this exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on two index cards or a sheet of paper, as announced in class and at the class website.

There are some trig function values given on page 3, in case you find them helpful on any of the problems.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)

Problem	Points	Score
1	16	
2	13	
3	14	
4	20	
5	18	
6	12	
7	20	
8	16	
TOTAL	129	

Problem 1 (16 points)Find the derivative $\frac{dy}{dx}$ for each of the following functions:

(a) $y = \ln(\sin(x))$

(b) $y = e^{x^2+2}$

(c) $y = (\tan^{-1} x)(\sin x)$

(equivalently, $y = (\arctan x)(\sin x)$)

(d) $y = \sin^{-1}(\sqrt{x})$

(equivalently, $y = \arcsin(\sqrt{x})$)

Problem 2 (13 points)

Using the definition of the derivative as a limit, find the derivative $\frac{df}{dx}$ for $f(x) = 3x^2 - 2x + 1$. (You will get no credit for writing down the derivative. You must show how it comes from the definition. When you take a limit, if you cancel or “plug in”, be sure to tell why that is valid.)

Here are a few trig function values in case they are helpful on any of the problems:

x	$\sin(x)$	$\cos(x)$	$\tan(x)$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	(undefined)

Problem 3 (14 points)

Let $f(x) = e^{\sin 2x}$.

(a) Find an equation for the tangent line to the graph of $f(x)$ at the point $(0, 1)$.

(b) Use the tangent line to find an approximate value for $f(0.1)$.

Problem 4 (20 points)

The function $f(x) = 3x^4 + 4x^3 - 12x^2$ has critical points where $f'(x) = 0$ at $x = -2$, $x = 0$, and $x = 1$. Find all absolute and local maxima and minima for $f(x)$ on the interval $-3 \leq x \leq 2$.

For each point you identify: (a) Tell whether it gives an absolute maximum, an absolute minimum, a relative maximum, and/or a relative minimum. (b) Tell how you know your answer to (a). (c) Give both the x -value and the resulting value of $f(x)$.

Problem 5 (18 points)

Evaluate the following integrals:

(a) $\int_{-1}^1 2x \cos(x^2 - 2) dx$

(b) $\int \frac{x - 2}{(x^2 - 4x + 3)^3} dx$

(c) $\int_0^{\frac{\pi}{2}} \sin^2(x) \cos(x) dx$

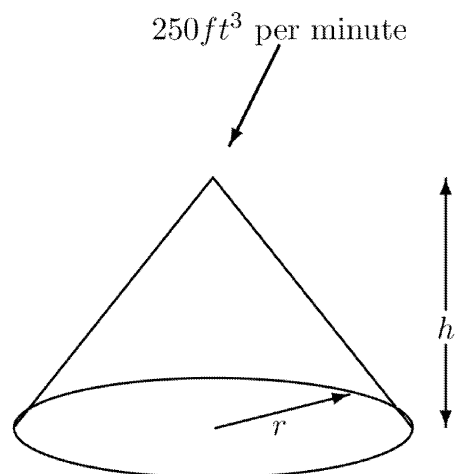
Problem 6 (12 points)

Solve the initial value problem: $\frac{dy}{dt} = \sin(t) + \cos(t)$, and $y\left(\frac{\pi}{2}\right) = 4$.

Problem 7 (20 points)

Sand is being added to a conical pile at a constant rate of 250 cubic feet per minute. The sand always flows out so that the height of the cone is the same as the radius of the circular base of the cone. At the instant when the radius is 5 feet, how fast is the radius increasing? (Remember the units!)

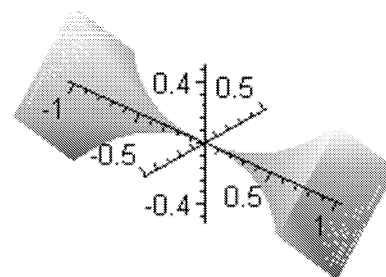
(The volume of a cone like this is $\frac{1}{3}$ times the height times the area of the base, i.e. $\frac{1}{3}h(\pi r^2)$.)



Problem 8 (16 points)

A solid object extends from $x = -1$ to $x = 1$. If we cut across the x -axis at any point in that range, the cross section is a square whose diagonal is $3x^2$. What is the volume of this object?

(If a square has sides of length s then its diagonal will be $\sqrt{2}s$.)





SCRATCH PAPER