**Your Name:** ______________________________

**Circle your TA's name:**

**Li Wang** | **Seth Meyer**

---

**Mathematics 221, Summer 2009** **(Wilson)**

## Midterm Exam July 9, 2009

**Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.**

**Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!**

**There is a problem on the back of this sheet: Be sure not to skip over it by accident!**

**There is scratch paper at the end of this exam. If you need more scratch paper, please ask for it.**

**You may refer to notes you have brought in on an index card or half a sheet of paper, as announced in class and at the class website.**

**BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)**

| Problem | Points | Score |
|---|---|---|
| 1 | 12 | |
| 2 | 12 | |
| 3 | 16 | |
| 4 | 12 | |
| 5 | 12 | |
| 6 | 12 | |
| 7 | 12 | |
| 8 | 12 | |
| TOTAL | 100 | |

Problem 1 (12 points)
Suppose $f(t) = t^3 + 3t^2 - 72t + 4$ gives the position, in inches along some scale, of an object, at time $t$ in seconds.

(a) What is the average rate of change of $f(t)$ as $t$ goes from $t = -1$ to $t = 1$? (Remember units!)

(b) What is the instantaneous rate of change of $f(t)$ at $t = 0$? (Remember units!)

(c) Find all local maxima and minima of $f(t)$. Be sure to indicate whether a given point gives a maximum or gives a minimum.

(d) $f(t)$ would not have to have absolute minimum and maximum values on the interval $-10 < t < 10$. Give an example of an interval on which you know $f(t)$ would have to have absolute maximum and minimum values, and tell why you know that.

Problem 2 (12 points)

Evaluate the derivatives $f'(x)$ for:

(a) $f(x) = 3x^2 + \frac{4}{x^3}$.

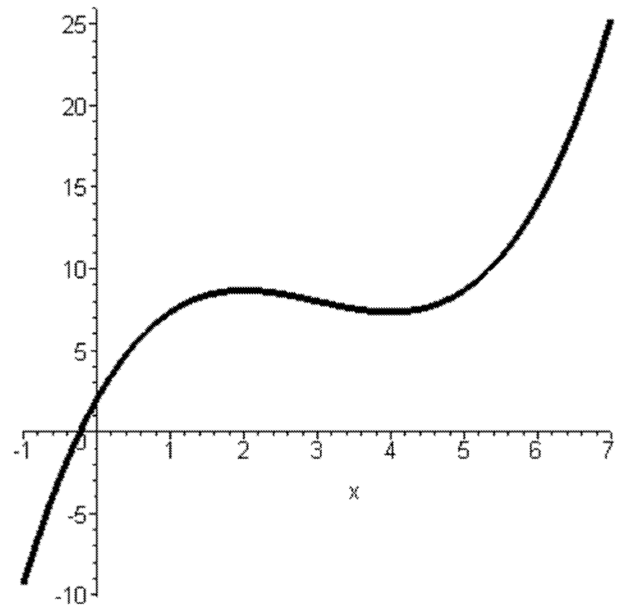
(b) $f(x) = \tan(3x^2 + 2)$.

(c) $f(x) = (x^2 + 1)\cos(4x - 1)$.

(d) $f(x) = \dfrac{x^2 \sin(x)}{\tan(x)}$.

Problem 3 (16 points)

The picture at the right shows part of the graph of a continuous function $f(x)$. Assume that if we continued the graph to the left and the right no "new" behavior would be found, that the slopes and curvatures continue from what you can see. Answer the questions below based on this function.




(a) $f'(x) = 0$ at $x =$
(List all numbers that make that statement true.)

(b) $f''(x) = 0$ at $x =$
(List all numbers that make that statement true.)

(c) On the interval $-\infty < x < 2.5$, $f''(x)$ is $< 0$ $= 0$ $> 0$. (Circle correct answer(s))

(d) At $x = 3$, $f(x)$ is $< 0$ $= 0$ $> 0$. (Circle correct answer(s))

(e) At $x = 3$, $f'(x)$ is $< 0$ $= 0$ $> 0$. (Circle correct answer(s))

(f) At $x = 3$, $f''(x)$ is $< 0$ $= 0$ $> 0$. (Circle correct answer(s))

(g) On the interval $2 < x < 4$, $f(x)$ is $< 0$ $= 0$ $> 0$. (Circle correct answer(s))

(h) On the interval $5 < x < 6$, $f'(x)$ is $< 0$ $= 0$ $> 0$. (Circle correct answer(s))

Problem 4 (12 points)

(a) Find an equation for the tangent line to the graph of $y = \tan(x)$ at the point $(\frac{\pi}{4}, 1)$.

(You may find helpful $\sin(\frac{\pi}{4}) = \cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.)

(b) Use a linear approximation to find an approximate value for $\tan(\frac{\pi}{4} - 0.1)$.

Problem 5 (12 points)
A metal can is to be made in the form of a right circular cylinder, to hold exactly one liter (1000 cubic centimeters) of some product. What are the dimensions (radius, height) in centimeters the cylinder should have in order to minimize the amount of metal required? (The amount is the area, we will assume the thickness has already been determined.)

(The volume in a cylinder is the area of one end times the height. The area in a circle of radius $r$ is $\pi r^2$. The outside of the cylinder is a rolled-up rectangle, and the area of a rectangle is its length times its width.)

Problem 6 (12 points)
Use the definition of the derivative as a limit to find $f'(x)$, for

$$f(x) = 3x^2 + 5x + 2.$$

(You could check your answer using formulas we have now for computing derivatives, but to get credit you must show how to find the answer using one of the forms of the definition as a limit that we have had.)

Problem 7 (12 points)

If $x$ and $y$ satisfy $xy^2 + x^2y = \sin(y)$, find $\dfrac{dy}{dx}$.

Problem 8 (12 points)

Use the $\epsilon$–$\delta$ definition of $\lim_{x \to a} f(x)$ to justify the statement:

$$\lim_{x \to 3}(4x - 2) = 10$$

Hint: You could use $\delta = \frac{\epsilon}{4}$, or other values, but the real point you need to address is why your choice of $\delta$ "works", i.e. meets the requirements of the definition.


SCRATCH PAPER