Your Name: _____

Mathematics 221, Spring 2006 Lecture 3 (Wilson)

Second Midterm Exam April 7, 2006

Write your answers to the six problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is a problem on the back of this sheet: Be sure not to skip over it by accident!

There is scratch paper at the end of this exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on up to three 3x5" index cards or one sheet of notebook paper, as announced in class.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)

Problem 1 (15 points)
Let $f(x) = 3x^4 - 4x^3 - 12x^2$, for $\infty \leq x \leq \infty$.

(a) Find all critical points of f.

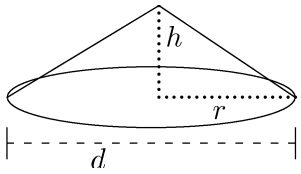
(b) On what interval(s) of numbers is $f(x)$ increasing? On what interval(s) of numbers is $f(x)$ decreasing?

(c) Find all (local or global) maxima and minima of f. Be sure to identify each point as to local or global, maximum or minimum.

Problem 2 (15 points)

Sand is pouring from a pipe at the rate of 16 cubic feet per second. If the falling sand forms a conical pile on the ground whose height h is always $\frac{1}{4}$ the diameter of the base of the pile, how fast is the height increasing when the pile is 4 feet high?

Hint: The volume of the pile is $\frac{1}{3}\pi r^2 h$ where r is the radius of the bottom of the pile.



Problem 3 (20 points)
Evaluate the integrals:

(a) $$\int_0^2 \frac{x^2}{(9-x^3)^{\frac{3}{2}}}\,dx$$

(b) $$\int 2x\sin(x^2)\,dx$$

Problem 4 (18 points)

(a) If $f'(x) = 3x^2 - 4x - \cos(x)$ and $f(0) = 3$, what is the function $f(x)$?

(b) (This f has nothing to do with part (a)!)
Let $f(x) = 3x^2 - 2$.

(i) Find a value c in $[0, 2]$ satisfying the Mean Value Theorem for integrals, i.e. $f(c)(2-0) = \int_0^2 f(x)\,dx$.

(ii) Find a value c in $[0, 2]$ satisfying the Mean Value Theorem for derivatives, i.e. $f'(c) = \dfrac{f(2) - f(0)}{2 - 0}$.

Problem 5 (18 points)

(a) What is the average value of $3x^2 - 2x + 1$ on the interval $[1, 4]$?

(b) Find the derivative $G'(x)$ if $G(x) = \int_3^{x^2} \sin(t)\, dt$.

Problem 6 (14 points)

Use derivatives (differentials) to find approximately the value $\sqrt[3]{7.9}$.

(The value from a calculator is ≈ 1.99163170129. You can use this in checking your answer but to get any credit you must use calculus to find your answer.)

SCRATCH PAPER