

Your Name: \_\_\_\_\_

Mathematics 221, Spring 2006

Lecture 3 (Wilson)

First Midterm Exam February 17, 2006

Write your answers to the six problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using  $\frac{\pi}{3}$ ,  $\sqrt{3}$ ,  $\cos(0.6)$ , and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is a problem on the back of this sheet: Be sure not to skip over it by accident!

There is scratch paper at the end of this exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on 3x5" index cards, as announced in class.

**BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)**

Problem	Points	Score
1	18	
2	18	
3	15	
4	15	
5	16	
6	18	
TOTAL	100	

Problem 1 (18 points)

For each of these functions  $f(x)$ , find  $f'(x)$ :

(a)  $f(x) = 5x^8 \tan(x)$

(b)  $f(x) = \frac{3x^2 - 2x + 1}{x^2 - 5}$

(c)  $f(x) = \sin(\cos(x^2))$

**Problem 2** (18 points)

Evaluate the limits, or tell if they don't exist:

(a)  $\lim_{x \rightarrow 0} \frac{\sin(x)}{2x}$

(b)  $\lim_{x \rightarrow 3} \frac{x^2}{x^2 - 9}$

(c)  $\lim_{x \rightarrow 3} \frac{x^2}{(x - 9)^2}$

Problem 3 (15 points)

For the function  $f(x) = 3x^2 - 4x + 1$ :

(a) What is the average rate of change of  $f(x)$  as  $x$  goes from 0 to 2?

(b) What is the derivative of  $f(x)$  as a function?

(c) What is the instantaneous rate of change of  $f(x)$  at  $x = 0$ ?

(d) What is the instantaneous rate of change of  $f(x)$  at  $x = 1$ ?

(e) What is the instantaneous rate of change of  $f(x)$  at  $x = 2$ ?

Problem 4 (15 points)

Use the definition of the derivative as a limit to find  $f'(x)$  for  $f(x) = 2x^2 + x + 3$ . (Just finding the derivative using formulas such as the power rule will get no credit in this problem.)

Problem 5 (16 points)

For  $f(x) = \sin(2x) + 3$ :

(a) Find an equation for the tangent line to the graph of  $f(x)$ , at the point  $(0, 3)$  on the graph.

(b) What is the second derivative  $f''(x)$ ?

(c) What is the second derivative  $f''(\frac{\pi}{4})$ ?

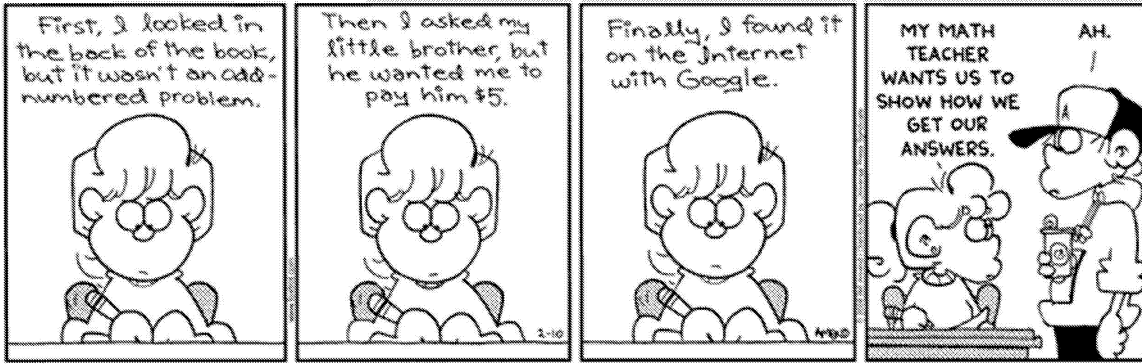
Problem 6 (18 points)

- (a) For  $\frac{3x-1}{x+3}$ , what is  $\lim_{x \rightarrow 1} f(x)$ ?

Give reasons for your answer, i.e. don't just give a number. Your reasons should relate to theorems we have studied: You do not need to cite theorems by number, but you do need to tell in some form what they say.

- (b) For  $f(x) = x^2 - 2$ : Show that there must be some value of  $x$  for which  $f(x) = 0$ , i.e. "there is a square root of 2".

Don't just give some numeric approximation to  $\sqrt{2}$ . Use some theorem(s) we have studied.



# SCRATCH PAPER