Mathematics 221, Lecture 1 Your Name: ____________________

Lecturer: Wilson TA's Name: ____________________

Final Exam 12/22/92

Write your answers to the 12 problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure to label which problem you are answering. Also be sure to circle your final answer. You have two hours for this examination. You may refer to notes you have brought in on up to three 4" by 6" index cards, as announced in class.

Wherever possible, leave your answers in exact forms (using π, $\sqrt{2}$, and similar numbers) rather than using decimal approximations.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	35	
2	20	
3	12	
4	15	
5	25	
6	15	
7	13	
8	10	
9	15	
10	13	
11	12	
12	15	
TOTAL	200	

Problem 1 (35 points)

(a) Find $\frac{dy}{dx}$ for

$$y = \ln \frac{(x^3+1)^5(x-1)^{\frac{3}{2}}}{\sqrt{(x^2+2x+5)}}$$

(b) Find $\frac{dy}{dx}$ for

$$y = \sin^{-1}(\sqrt{x})$$

(c) Find $\frac{dy}{dx}$ for

$$y = \int_0^{x^3} \cos(t)dt$$

(d) The function defined by $\int_\pi^{x^3} \cos(t)dt$ has the same derivative as the function y in part (c): In one or two sentences tell why that should be true.

(e) Find $\frac{dy}{dx}$ for

$$y = x \ln(\sin^{-1}(x))$$

(f) Find $\frac{dy}{dx}$ for

$$y = (3x^2 + 2)^x$$

(g) Find $\frac{dy}{dx}$ for

$$y = \log_3(x^2 + 1)$$

Problem 2 (20 points)
Find the following limits, or for any which do not exist tell why that is the case:

$$(a) \quad \lim_{x \to 0} \frac{x^2 + 2x - 3}{x^2 - 1}$$

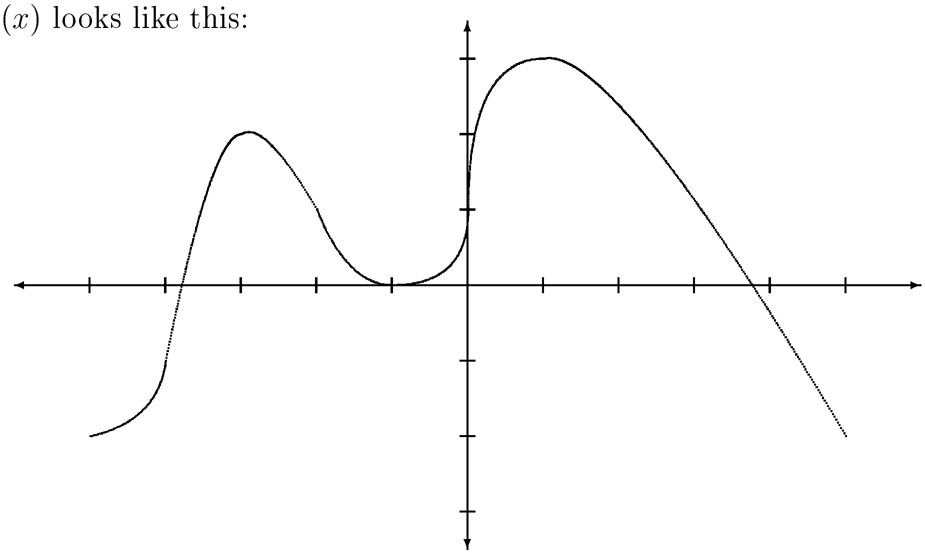
$$(b) \quad \lim_{x \to 1} \frac{x^2 + 2x - 3}{x^2 - 1}$$

$$(c) \quad \lim_{x \to \infty} \frac{x^2 + 2x - 3}{x^2 - 1}$$

$$(d) \quad \lim_{x \to \infty} \frac{x^2 + 2x + 5}{e^x}$$

$$(e) \quad \lim_{x \to \infty} \sin(x)$$

Problem 3 (12 points)
The graph of $y(x)$ looks like this:



Using

A for the interval $[-5,-3]$
B for the interval $[-4,-2]$
C for the interval $[-4,-1]$
D for the interval $[-3,-1]$
E for the interval $[-3,1]$
F for the interval $[-1,1]$
G for the interval $[0,5]$

H for the interval $[1,5]$
I for the point $(-4,-1)$
J for the point $(-3,2)$
K for the point $(-2,1)$
L for the point $(0,1)$
M for the point $(1,3)$
N for the point $(3,1)$

fill in one (or more) of the letters A - N in the blank in each of parts (a) - (f) below to make the statement true: You should fill in *all* of the letters which apply in each blank.

(a) The graph is *increasing* on the interval(s) ________________________

(b) The graph is *decreasing* on the interval(s) ________________________

(c) The graph has a *local maximum* at the point(s) ________________________

(d) The graph has an *absolute maximum* at the point(s) ________________________

(e) The graph is *concave downward* on the interval(s) ________________________

(f) The graph is has an *inflection point* at the point(s) ________________________

Problem 4 (15 points)

Two cars are traveling on straight roads which intersect at right angles. Car A is going west at 40 miles *per* hour and car B is going north at 50 miles *per* hour. At the instant when car A has 10 miles to go before reaching the intersection and car B has gone 24 miles beyond the intersection, how fast is the distance between the cars changing? Is it increasing or decreasing?

Problem 5 (25 points)
Evaluate the following integrals:

$(a) \quad \int \frac{1+e^{2x}}{e^x} dx$

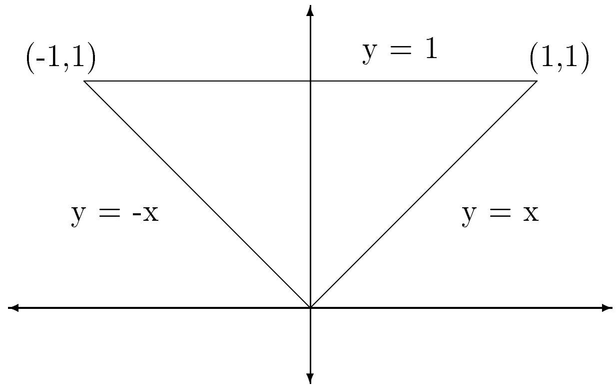
$(b) \quad \int \frac{e^x}{1+e^{2x}} dx$

$(c) \quad \int \frac{e^{2x}}{1+e^{2x}} dx$

$(d) \quad \int_0^{\frac{\sqrt{2}}{2}} \frac{2x}{\sqrt{1-x^4}} dx$

$(e) \quad \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sec(x)\tan(x) dx$

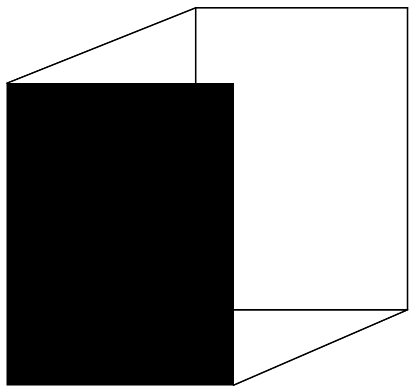
Problem 6 (15 points) A triangular plate of thin metal covers the triangle whose boundaries are the lines $y = x$, $y = -x$, and $y = 1$. (see picture) The plate has density $\delta(x, y) = x^2$. Find the center of mass of the plate.



Problem 7 (13 points)
Find the area of the region between the curves $y = -x^2 + 3x + 10$ and $y = 2x - 2$.

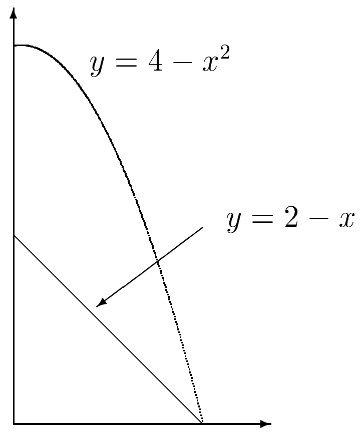
Problem 8 (10 points)
Set up but *DO NOT EVALUATE* an integral to compute the arc length of the section of the curve $y = x + \sin(x)$ between $(0, 0)$ and (π, π).

Problem 9 (15 points)
A storage bin is to be made from 108 square inches of sheet metal. The bin will be in the form of a box with the top and one side missing. The bottom of the box will be square. Find the dimensions which give the largest volume inside the bin. (In the picture below the *outside* of the bin has been painted black to make it easier to see which part is in front.)



Problem 10 (13 points)

The region between the curves $y = 4 - x^2$ and $y = 2 - x$ for $x \geq 0$ is rotated about the x-axis. Set up but *DO NOT EVALUATE* an integral to find the volume of the resulting solid.



Problem 11 (12 points)
The population of a colony of bacteria at time t is given by $P(t)$, and the growth rate of the population is proportional to the value of the population. If the population a time $t = 0$ is 100,000 and the population at time $t = 10$ is approximately 738,906, find a formula for $P(t)$.
(Note: You might find useful the fact that e^2 is approximately 7.389056 .)

Problem 12 (15 points)
An object moves along the x-axis so that its acceleration at time t is given by

$$a(t) = \sin(x) + e^{-x}$$

If the object starts at $s(0) = 0$ at time $t = 0$, with velocity $v(0) = 3$, find the position and velocity as functions of t.

SCRATCH PAPER