

## Exam II 10/26/92

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure to label which problem you are answering. Also be sure to circle your final answer.

Wherever possible, leave your answers in exact forms (using  $\pi$ ,  $\sqrt{2}$ , and similar numbers) rather than using decimal approximations.

You may refer to notes you have brought in on one 4" by 6" index card, as announced in class.

**BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.**

Problem	Points	Score
1	14	
2	12	
3	11	
4	12	
5	14	
6	13	
7	8	
8	16	
TOTAL	100	

Problem 1 (14 points)

(a) Suppose that the functions  $f(x)$  and  $g(x)$  and their derivatives with respect to  $x$  have the following values at  $x = 0$  and  $x = 1$ :

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	1	5	$\frac{1}{3}$
1	3	-4	$-\frac{1}{3}$	$-\frac{8}{3}$

Find the derivatives with respect to  $x$ , at  $x = 0$ , of

(i)  $f(g(x))$

(ii)  $g(f(x))$

(b) Given that variables  $x$  and  $y$  satisfy the relation  $x^2y^3 - y^2 = 1 + 2x$ , find  $\frac{dy}{dx}$  as a function of  $x$  and  $y$ .

(c) Find  $\frac{dy}{dx}$  if  $y = \sec^3(x^2 + 1)$ .

Problem 2 (12 points) Find the following limits, if they exist:

(i)

$$\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1}$$

(ii)

$$\lim_{x \rightarrow \infty} \frac{x + \sin x}{x + 1}$$

(iii)

$$\lim_{x \rightarrow \frac{\pi}{2}} \left( \frac{2x}{\cos x} - \pi \sec x \right)$$

(iv)

$$\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x + 1}$$

Problem 3 (11 points)

A boat sails parallel to a straight beach at a velocity of 15 miles *per* hour. It stays 4 miles offshore. It is approaching a lighthouse which is on the shoreline. At the instant when the distance from the boat to the lighthouse is 5 miles, how fast is the boat approaching the lighthouse? (Your answer should include units.)

Problem 4 (12 points)

You are to design a container which has the form of a right circular cylinder with one end closed and one open. The container has to hold  $512\pi$  cubic inches. Find the dimensions which give the least surface area. (Ignore waste considerations: Only the area of the material actually used in the container is to be minimized. Remember that the container has one end open.)

(The area of a rectangle with sides  $x$  and  $y$  is  $xy$ . The circumference of a circle of radius  $r$  is  $2\pi r$ . The area of a circular disk of radius  $r$  is  $\pi r^2$ . The volume of a cylinder is the area of the base times the height.)

Problem 5 (14 points)

(a) Let  $f(x) = \sqrt[4]{x}$ . Choose an appropriate whole number  $a$  and:

(i) Find the linearization of  $f$  at  $a$ .

(ii) Use this linearization to calculate a good approximation to  $\sqrt[4]{17}$ . (Show the answer to your approximation as an expression *without* decimal fractions: *E.g.*  $3\frac{1}{40}$  would be an acceptable *form* for the answer, although it is not the right answer, and 3.025 would not be a valid answer.)

(b) Find a polynomial  $g(x)$  (with only whole number powers in it, *i.e.* no radicals or fractional exponents) such that using  $\sqrt[4]{17}$  for  $x$  will make  $g(x) = 0$ . Using an initial guess of 2 for the value of  $x$  which makes  $g(x) = 0$ , apply Newton's method once to compute an improved approximation for that value.

Problem 6 (13 points)

Find all maxima and minima of  $y = x^3 - x^2 + 1$  on the interval  $[-1, 1]$ . Give the  $x$  values where these occur, and for each tell whether it gives a local maximum, an absolute (global) maximum, a local minimum, or an absolute (global) minimum. (A point may be more than one of these. Give all that apply.)

Problem 7 (8 points)

Find all asymptotes (horizontal, vertical, or oblique) of

$$y = \frac{6x^2 + x + 1}{3x + 2}$$

Give your answers as equations for straight lines.



Problem 8 (16 points)

$f(x)$  is a function defined for every real number *except*  $x = -1$  and  $x = 4$ . We know the following facts about  $f$ :

(a)

$$\lim_{x \rightarrow -\infty} f(x) = 2.$$

(b)

$$\lim_{x \rightarrow +\infty} f(x) = 1.$$

(c)  $f$  has vertical asymptotes  $x = -1$  and  $x = 4$ .

(d)  $f'$  is:

positive on  $(-\infty, -1)$

positive on  $(-1, 1)$

negative on  $(1, 3)$

positive on  $(3, 4)$

negative on  $(4, \infty)$

(e)  $f''$  is:

positive on  $(-\infty, -1)$

negative on  $(-1, 2)$

positive on  $(2, 4)$

positive on  $(4, \infty)$

(f)  $f$  takes on the values:

$$f(1) = 1$$

$$f(2) = -1$$

$$f(3) = -2$$

Sketch a graph of  $f(x)$ . While artistic talent is not required, your graph *should* exhibit all features listed above.

