Mathematics 221, Lecture 5 (Wilson) Your Name: _____

Circle your TA's name:

Joni Baker Brian Curtin Cheryl Grood Junmin Gu

Javier Medina Randy Pruim Jennifer Ziebarth

Exam IB

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.
Wherever applicable, leave your answers in exact forms (using π, $\sqrt{3}$, and similar numbers) rather than using decimal approximations. There is scratch paper at the end of the exam. If you need more scratch paper, please ask for it.
You may refer to notes you have brought in on one sheet of paper (regular notebook or typing size) as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. IF YOUR ANSWER WAS OBTAINED FROM A CALCULATOR (MORE THAN JUST USING THE CALCULATOR TO EVALUATE A NUMERIC EXPRESSION) EXPLAIN IN DETAIL HOW THE CALCULATOR WAS USED TO OBTAIN IT.

Problem	Points	Score
1	10	
2	16	
3	12	
4	15	
5	12	
6	13	
7	12	
8	10	
TOTAL	100	

Problem 1 (10 points)
Use the definition of the derivative as a limit to find $f'(x)$ for

$$f(x) = -2x^2 + 3x - 1$$

Problem 2 (16 points)

(a) Let

$$f(x) = \frac{3}{(x-1)(x+2)^2}$$

For what real numbers is f continuous?

Find the following limits (or tell why they don't exist):

$$\lim_{x \to \infty} f(x)$$

$$\lim_{x \to 2} f(x)$$

$$\lim_{x \to 1} f(x)$$

(b) Let

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < -1 \\ 2x + 2 & \text{if } -1 < x \leq 2 \\ 10 - x^2 & \text{if } 2 < x \end{cases}$$

Is $f(x)$ continuous at $x = -1$? (give reasons!)

Is $f(x)$ continuous at $x = 2$? (give reasons!)

Problem 3 (12 points)

An object moves vertically such that its height (in feet) at time t (in seconds) is given by

$$h(t) = 1 + \cos \pi t$$

for $0 \leq t \leq 4$.

(a) For what value(s) of t is the object at its highest point, and what is the height at those times?

(b) For what value(s) of t is the object at its lowest point, and what is the height at those times?

(c) How fast is the object moving when:

$t = 1$?

$t = 1.5$?

$t = 3$?

(d) What is the average velocity of the object between the times when $t = 2$ and $t = 2.5$?

Problem 4 (15 points)

(a) If $f(x) = x^3(x^4 + 2x - 3)$, what is $\frac{df}{dx}$?

(b) If $f(x) = \tan^2 x + 1$, what is $\frac{df}{dx}$?

(c) If $f(x) = \frac{\sin x}{x^2-5}$, what is $\frac{df}{dx}$ at $x = \pi$?

(d) If $f(x) = \arctan(\cos(x))$, what is $\frac{df}{dx}$?

(e) If $f(x) = \frac{3x^2-3}{2x+1}$, what is $\frac{df}{dx}$ at $x = 2$?

Problem 5 (12 points)

Suppose $f(x)$ is some function such that $f'(x) = x \sin(\pi x)$.

Let $g(x) = f(x^2)$.

(a) What is $g'(x)$?

(b) What is $g'(1.5)$?

(c) What is $g''(x)$?

Problem 6 (13 points)
Suppose variables x and y satisfy the relation

$$xy + 2y^2 \cos x = 8$$

Find equations for the tangent and normal lines to the graph of $f(x)$ at the point $(0, 2)$. Be sure to make clear which equation gives the tangent and which gives the normal!

Problem 7 (12 points)

Of the eight functions graphed below, four are the derivatives of the other four. Fill in the letters A-H in the blanks, and give reasons.

Function _____ can only be the derivative of function _____, because:

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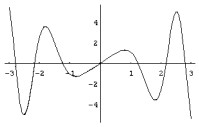
Function _____ can only be the derivative of function _____, because:

A



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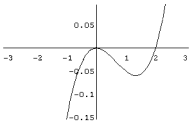

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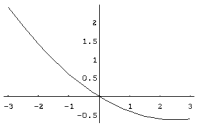

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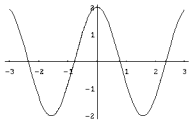

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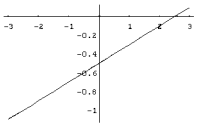

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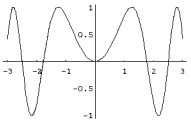

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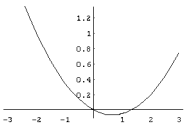

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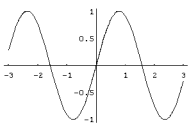

Problem 8 (10 points)
Find approximate values for $\sqrt[5]{30}$ in two ways:

(a) Use Newton's method to find a solution of $x^5 - 30 = 0$:

For this equation, show how to calculate the next approximation x_{n+1} from the previous x_n.
(Not just the general formula as it would be applied to all equations!)

Beginning with $x_0 = 2$, find x_1.

(b) Use the standard linear approximation to $f(x) = \sqrt[5]{x}$ centered at $a = 32$.

SCRATCH PAPER