Your Name: _____

Circle your TA's name:

George Brown | Carl Carlson | David Milovich

Yuanqi Wang | Michael Woodbury | Fang Yuan

Mathematics 221, Fall 2007 — Lecture 1 (Wilson)

Final Exam December 17, 2007

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is a problem on the back of this sheet: Be sure not to skip over it by accident!

There is scratch paper at the end of this exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in, as announced in class and at the class website.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)

Problem	Points	Score
1	30	
2	25	
3	25	
4	24	
5	30	
6	20	
7	26	
8	20	
TOTAL	200	

Problem 1 (30 points)

Find the derivative $\frac{dy}{dx}$ for:

(a) $y = \int_1^{x^2} \sin(3t^2 - 2)\, dt$

(b) $y = \sin^{-1}(3x^2)$

(c) $y = \left(e^{x^2}\right) \ln(x)$

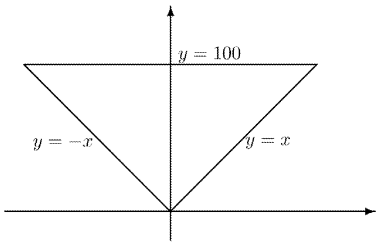
Problem 2 (25 points)

Find all maximum and minimum points for the function $f(x) = x^3 - 3x^2 - 9x + 2$ on the closed interval $-4 \leq x \leq 3$.

For each point you select, tell whether it is a maximum or a minimum, whether it is local or global, and how you know which of each of these it is. Be sure to give both the x value and the value of f at each point.

Problem 3 (25 points)

A dam in the form of an isoceles right triangle has for its horizontal top edge the hypotenuse of the triangle. It stands 100 feet high. You can picture it in relation to a set of axes in the diagram to the right.



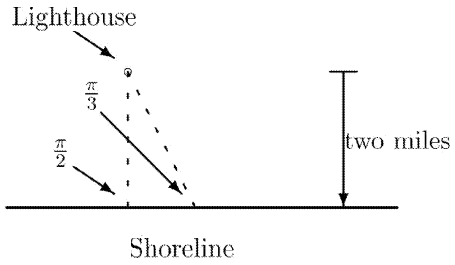
Water weighing 62.4 pounds per cubic foot stands against one side of the dam from the bottom all the way to the top. That produces a pressure against the dam at a distance h feet down from the top that is $62.4\,h$ pounds per square foot, 0 at the top and 6240 pounds per square foot at the bottom point (where $y = 0$ for the axes in the picture).

Set up but do not evaluate an integral that would compute the total force agains the dam resulting from the water.

Describe how you get your integral: Don't just write down an answer. Explain your steps.

Problem 4 (24 points)

A lighthouse 2 miles from a straight shoreline rotates counter-clockwise (viewed from above) once per minute. The lighthouse sends out a beam of light, which is thus rotating at 2π radians per minute. The picture at the right shows the situation at the two instants that matter in the questions below.



(a) At an instant when the light beam is aimed straight toward the shoreline, so it hits the shore at a right angle, how fast in miles/minute is the spot where the beam hits the shore moving?

(b) Slightly later, when the beam is at $\frac{\pi}{3}$ radians to the shore rather than $\frac{\pi}{2}$, how fast is the lighted spot moving?

Problem 5 (30 points)

Evaluate the integrals. (Notice that some are definite and some indefinite.)

(a) $\displaystyle\int \sin^5\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)\,dx$

(b) $\displaystyle\int_0^1 \left(\sqrt{t^5+2t}\right)\,(5t^4+2)\,dt$

(c) $\displaystyle\int \frac{dx}{2+(x-1)^2}$

Problem 6 (20 points)

Suppose x and y are quantities varying with time t, and that at all times they are related by $x = \sin(y)$. (Note that in that case, at any time t, we would have $-1 \leq x \leq 1$.)
Suppose we also know that at any time, $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.

(a) Write a formula giving y as a function of x.

(b) What is $\frac{dy}{dx}$?

(c) If $\frac{dx}{dt} = 3$ and $x = \frac{\sqrt{2}}{2}$ and $y = \frac{\pi}{4}$, what is $\frac{dy}{dt}$?

Problem 7 (26 points)

(a) Find the total area between the graph of $f(x) = x^2 - 2x$ and the x-axis, for $1 \leq x \leq 3$.

(b) A function $g(x)$ has its derivative proportional to the function itself, i.e. $g'(x) = k\,g(x)$ for some constant k. Find a formula for $g(x)$ if $g(0) = 3$ and $g'(0) = 6$.

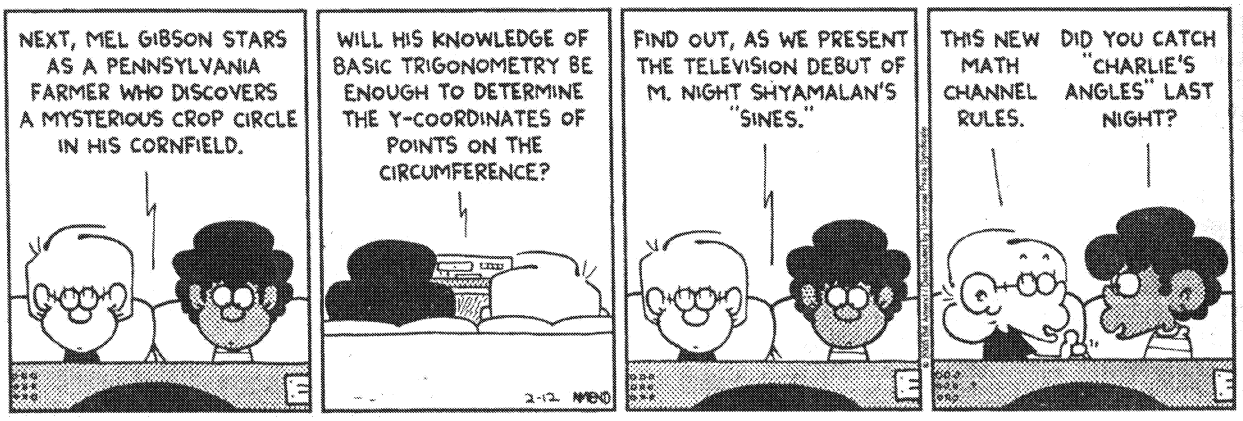
Problem 8 (20 points)

(a) Find an equation for the tangent line to the graph of $y = \cosh(x)$ at $(\ln 2, \frac{5}{4})$.

(b) Assume $\ln 2 = 0.69$, which is approximately right. Use the linearization of $\cosh(x)$ at $x = \ln 2$ to give an approximation to $\cosh(0.68)$.

(Make use of the fact that $0.68 = 0.69 - 0.01$.)

FOXTROT



SCRATCH PAPER

{not a command!}